



The Tuition Centre Pre-Leaving Certificate Exam, 2020

Mathematics

Paper 2

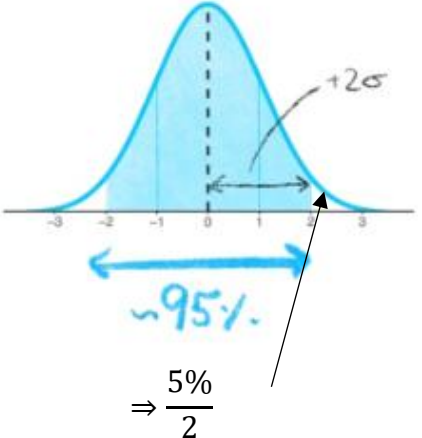
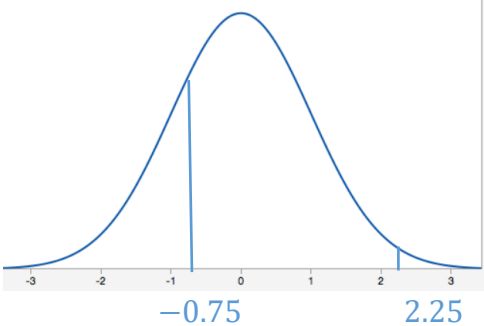
Higher Level

Time: 2 hours and 30 minutes

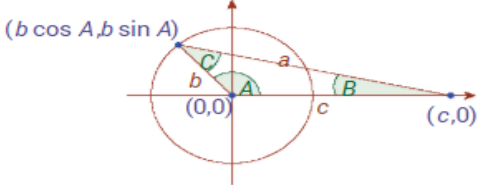
300 Marks

Q1	Model Solution – 25 Marks	Marking Notes
(a)	<p>Labels & 0.03 Labels & 0.7,0.02</p>	<p>MS (0, 3, 4, 5)</p> <p>LPC: Some work of merit eg any correct probabilities given on the tree diagram</p> <p>HPC: Fully correct tree diagram with one or two mistakes</p>
(b)	$P(\text{Exactly one defect}) = 0.03 \times 0.3 + 0.97 \times 0.02$ $= 0.009 + 0.0194$ $= 0.0284$	<p>MS (0, 3, 4, 5)</p> <p>LPC: Work of merit to find answer</p> <p>HPC: One error in calculation</p>
(c)	$P(\text{No defects}) = (0.97) \times (0.98) \times (1 - 0.05)$ $= 0.90307$	<p>MS (0, 3, 4, 5)</p> <p>LPC: Work of merit to find answer</p> <p>HPC: One error in calculation</p> <p>**4 marks for answer rounded incorrectly**</p>
(d)	$P(\text{Exactly one defect}) =$ $= (0.97 \times 0.02 \times 0.95) + (0.03 \times 0.3 \times 0.95) + (0.97 \times 0.98 \times 0.05)$ $= 0.07451$	<p>MS (0, 4, 6, 8, 10)</p> <p>LPC: Work of merit eg one bracket correct</p> <p>MPC: Two elements correct</p> <p>HPC: Correct method but 1 error in calculation</p> <p>**9 marks for answer rounded incorrectly**</p>

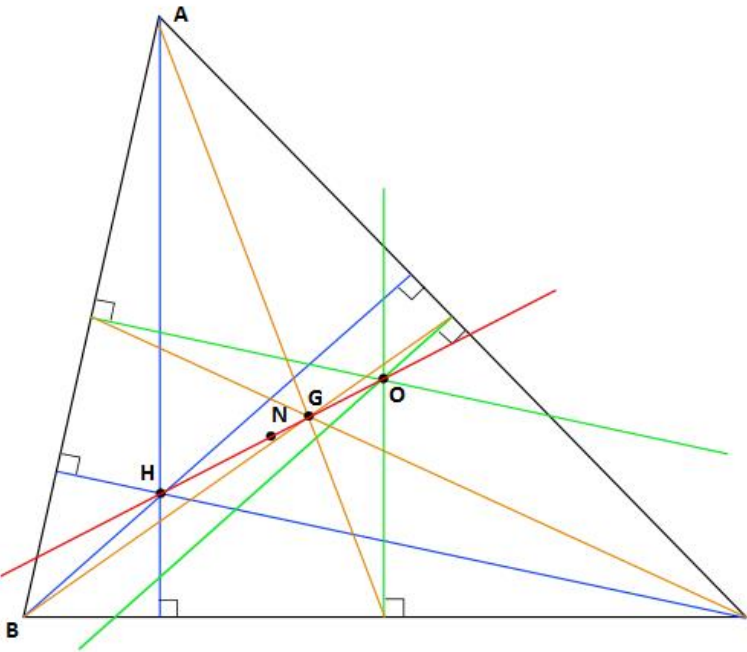
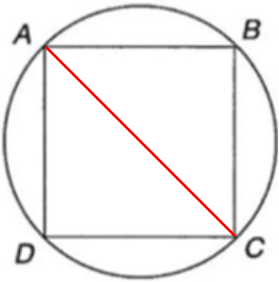
Q2	Model Solution – 25 Marks	Marking Notes
(a)	$x^2 + y^2 + 2gx + 2fy + c = 0$ $x^2 + y^2 - 6x + 4y = 12$ $2g = -6 \quad 2f = 4$ $-g = 3 \quad -f = 2$ <p style="text-align: center;">Centre: (3, -2)</p> $\text{Radius} = \sqrt{g^2 + f^2 - c}$ $= \sqrt{3^2 + (-2)^2 - (-12)}$ $= \sqrt{25} = 5$ $(x - 3)^2 - 9 + (y + 2)^2 - 4 = 12$ $(x - 3)^2 + (y + 2)^2 = 12 + 9 + 4$ <p style="text-align: center;">Centre: (3, -2)</p> $r = \sqrt{25} = 5$	<p>MS (0, 3, 5)</p> <p>PC: Either centre or radius correct or work of merit for both</p>
(b)	$PQ = \sqrt{(7 - -1)^2 + (-5 - 1)^2}$ $PQ = \sqrt{(8)^2 + (6)^2}$ $PQ = 10 = 2 \times \text{radius} \therefore \text{diameter}$ <p style="text-align: center;">OR</p> $MP \text{ of } PQ = \left(\frac{7 + -1}{2} \right), \left(\frac{1 + -5}{2} \right) = (3, -2)$	<p>MS (0, 4, 8,10)</p> <p>LPC: Work of merit</p> <p>HPC: Correct answer without conclusion (e.g. PQ =10 and stops)</p>
(c)	<p>R lies on C (angle on semi circle theorem)</p> $x = 0 \Rightarrow y^2 + 4y - 12 = 0$ $(y - 2)(y + 6) = 0$ $y = -6 \text{ or } y = 2$ <p style="text-align: center;">Correct answer (0,2)</p>	<p>MS (0, 4, 6, 8, 10)</p> <p>LPC: Work of merit to find answer eg marking R on the diagram</p> <p>MPC: Substitution of 0</p> <p>HPC: Correct equation in y</p>

Q3	Model Solution – 25 Marks	Marking Notes
(a)	<p>a) $520 + 2(80) = 680$ $\bar{x} + 2\sigma = 680$ $\bar{x} \pm 2\sigma = 95\%$ (Empirical Rule)</p>  <p>$\Rightarrow \frac{5\%}{2}$ $= 2.5\%$ purchased over 680 litres</p>	<p>MS (0, 4, 7, 10)</p> <p>LPC: Any relevant work. For example draws the normal distribution and labels the correct intervals. Or writes out the values of the empirical rule, 68%, 95% and 99.7%</p> <p>HPC: For correctly identifying 95%</p>
(b)	<p>a) $P(460 \leq X \leq 700)$</p> $z = \frac{x - \mu}{\sigma}$ <p>$x = 460$ $x = 700$ $z = \frac{460 - 520}{80}$ $z = \frac{700 - 520}{80}$ $= \frac{-60}{80}$ $= \frac{180}{80}$ $= -0.75$ $= 2.25$</p>  <p>$1 - (0.7734)$ 0.9878 $= 0.2266$</p> <p>$\Rightarrow P(460 \leq X \leq 700) = 0.9878 - 0.2266 = 0.7612$</p>	<p>MS (0, 4, 7, 10)</p> <p>LPC: any relevant step.</p> <p>HPC: finding $P(z$ is greater than or equal to $-0.75)$ or $P(z$ is less than or equal to $2.25)$ or both but then fails to finish.</p>

(c)	<p>a) $P(z \leq z_1) = 0.9$ $\Rightarrow z_1 \approx 1.28$ $\Rightarrow z_2 \approx -1.28$</p> $z = \frac{x - \mu}{\sigma}$ $-1.28 = \frac{x - 520}{80}$ $-1.28(80) = x - 520$ $-102.4 = x - 520$ $-102.4 + 520 = x$ $417.6 = x = \textit{number of litres}$ $\Rightarrow 417.6 \times \text{€}0.89$ $= \text{€}371.664$ $\approx \text{€}371.66$	<p>MS (0, 3, 4, 5)</p> <p>LPC: finding z score of 1.28 and stops.</p> <p>HPC: finding 417.6 but fails to multiply by €0.89c</p> <p>**4 marks for incorrect rounding**</p>
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Q4	Model Solution – 25 Marks	Marking Notes
(a)	 <p>Using the points $(c, 0)$ and $(b \cos A, b \sin A)$ And the distance formula:</p> $a = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $a = \sqrt{(b \cos A - c)^2 + (b \sin A - 0)^2} \text{ (square both sides)}$ $a^2 = (b \cos A - c)^2 + (b \sin A - 0)^2 \text{ (square brackets)}$ $a^2 = b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A$ $a^2 = b^2 (\cos^2 A + \sin^2 A) + c^2 - 2bc \cos A + c^2$ $a^2 = b^2 (1) + c^2 - 2bc \cos A + c^2$ $\therefore a^2 = b^2 + c^2 - 2bc \cos A$ $(\cos^2 A + \sin^2 A = 1)$	MS (0, 5, 10, 15) LPC: Work of merit eg any correct substitution into the distance formula HPC: 1 or 2 mistakes
(b)	$a^2 = b^2 + c^2 - 2bc \cos A \text{ (from part a)}$ $b^2 = a^2 + c^2 - 2ac \cos B \text{ (swap a and b from part a)}$ <p>Now subtract a^2 from b^2</p> $b^2 - a^2 = a^2 - b^2 - 2ac \cos B + 2bc \cos A \text{ HPC}$ $b^2 - a^2 - a^2 + b^2 = -2ac \cos B + 2bc \cos A$ $2b^2 - 2a^2 = -2ac \cos B + 2bc \cos A$ $2(b^2 - a^2) = 2bc \cos A - 2ac \cos B$ $b^2 - a^2 = c(b \cos A - a \cos B)$	MS (0, 4, 8, 10) LPC: Work of merit like rewriting the cosine rule as $b^2 = a^2 + c^2 - 2ac \cos B$ HPC: 1 or 2 mistakes

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$4y + 3 = 2x$ $y = \frac{2x - 3}{4}$ $m(L_1) = \frac{1}{2}$ $m(L_2) = -2$ $(L_2): y - 4 = -2(x - 2)$ $2x + y - 8 = 0$	<p>MS (0, 2, 3, 4, 5)</p> <p>LPC: attempt to find the slope of l_1</p> <p>MPC: Correctly finding the slope of l_2 from their slope of l_1</p> <p>HPC: Work of merit to find equation of line</p>
(b)	$y = 8 - 2x$ $4(8 - 2x) + 3 = 2x$ $x = 3.5, y = 1$	<p>MS (0, 3, 5)</p> <p>PC: Work of merit eg isolating y and substituting into the other equation</p>
(c)	$CD = \sqrt{(3.5 - 2)^2 + (1 - 4)^2}$ $= \sqrt{1.5^2 + 3^2}$ $= 1.5\sqrt{1^2 + 2^2}$ $= 1.5\sqrt{5}$ $= \frac{3}{2}\sqrt{5}$	<p>MS (0, 3, 5)</p> <p>PC: Work of merit like some correct substitution into the length of a line formula</p>
(d)	$\text{Area} = \text{triangle } ABC + \text{triangle } ABE$ $= \frac{1}{2} \times \frac{3}{2}\sqrt{5} \times \sqrt{80} + \frac{1}{2} \times 3\sqrt{5} \times \sqrt{80}$ $= \frac{3}{4}\sqrt{5} \times 4\sqrt{5} + \frac{3}{2}\sqrt{5} \times 4\sqrt{5}$ $= \frac{3}{4}(20) + \frac{3}{2}(20)$ $= 45$	<p>MS (0, 4, 7, 8, 10)</p> <p>LPC: Some correct substitution into the equation of a triangle formula</p> <p>MPC: Correctly find the area of one of the triangles</p> <p>HPC: Fully correct method and finds a final answer but with one mistake</p> <p>**9 marks for no units**</p>

Q6	Model Solution – 25 Marks	Marking Notes
(a)	 <p data-bbox="288 824 1139 846"><i>Blue lines are altitudes, orange medians, green perpendicular bisectors, and the red line is the Euler line</i></p>	<p data-bbox="1257 143 1469 215">MS (0, 5, 8, 12, 15)</p> <p data-bbox="1257 255 1501 286">LPC: One correct</p> <p data-bbox="1257 327 1417 398">MPC: Two correct</p> <p data-bbox="1257 439 1453 546">HPC: Three correct but not joined</p>
(b)	 $ ac ^2 = 3^2 + 3^2$ $ ac ^2 = 9 + 9$ $ ac ^2 = 18$ $ ac = 3\sqrt{2} = \text{diameter}$ $\Rightarrow \text{radius} = \frac{3\sqrt{2}}{2}$ $l = 2\pi r$ $l = 2\pi \frac{3\sqrt{2}}{2}$ $l = \sqrt{18}\pi / 3\sqrt{2}\pi / 13.2286 \dots$	<p data-bbox="1257 934 1469 965">MS (0, 4, 8, 10)</p> <p data-bbox="1257 1005 1485 1077">LPC Any use of Pythagoras</p> <p data-bbox="1257 1117 1485 1261">HPC Correct diameter and use of circumference formula</p> <p data-bbox="1257 1301 1465 1408">**9 marks for correct answer with no units**</p>

Q7	Model Solution – 50 Marks	Marking Notes
(a)		
(i)	$9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos \alpha$ $\cos \alpha = \frac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6} = -\frac{29}{48} = -0.604\dots$ $\alpha = 2.22 \text{ radians}$	<p>MS (0, 4, 8, 10)</p> <p>LPC: Work of merit eg some substitution into the cosine rule</p> <p>HPC: Correct use of cosine rule without getting the correct answer</p> <p>** 9 marks for answer rounded incorrectly **</p>
(ii)	$2\pi - 2.22 = 4.06366 \dots\dots$ $\text{Area} = \frac{1}{2} \times 4^2 \times 4.06$ $= 32.5 \text{ cm}^2$	<p>MS (0, 4, 8, 10)</p> <p>LPC: Work of merit eg finding the angle or substitution into the area formula</p> <p>HPC: Correct method with 1 mistake</p> <p>** 9 marks for rounding and or units error**</p>
(iii)	$\text{Area of triangle} = \frac{1}{2} \times 4 \times 6 \times \sin(2.22)$ $= 9.56$ $\text{Area required} = 9.56 + 32.5$ $\text{Area} = 42.1 \text{ cm}^2$	<p>MS (0, 4, 8, 10)</p> <p>LPC: Work of merit e.g. finding the area of the triangle</p> <p>HPC: Correct method with 1 mistake</p> <p>** 9 marks for rounding and or units error**</p>
(iv)	$\text{Arc length} = 4 \times 4.06 = 16.24$ $\text{Perimeter} = ZY + WY + \text{Arc length}$ $\text{Perimeter} = 27 \text{ cm}$	<p>MS (0, 4, 8, 10)</p> <p>LPC: Work of merit eg finding the arc length</p> <p>HPC: Correct method with 1 mistake</p>

(b)

$$\text{Using } \left(\frac{\pi}{10}, 0\right) \Rightarrow \sin\left(a\frac{\pi}{10} - b\right) = 0$$

$$\left(a\frac{\pi}{10} - b\right) = \sin^{-1}(0)$$

$$\left(a\frac{\pi}{10} - b\right) = 0$$

$$a\frac{\pi}{10} = b$$

$$\text{Using } \left(\frac{3\pi}{5}, 0\right) \Rightarrow \sin\left(a\frac{3\pi}{5} - b\right) = 0$$

$$\left(a\frac{3\pi}{5} - b\right) = \sin^{-1}(0)$$

$$\left(a\frac{3\pi}{5} - b\right) = \pi$$

$$a\frac{3\pi}{5} - \pi = b$$

$$\text{Equating } \Rightarrow a\frac{3\pi}{5} - \pi = a\frac{\pi}{10}$$

$$\frac{3a}{5} - 1 = \frac{a}{10}$$

$$\frac{-5a}{10} = -1$$

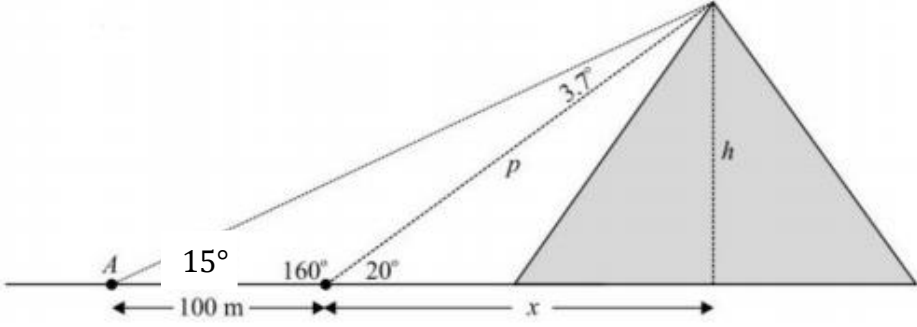
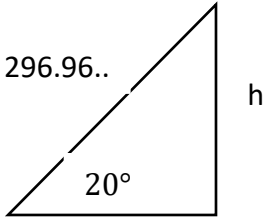
$$a = 2$$

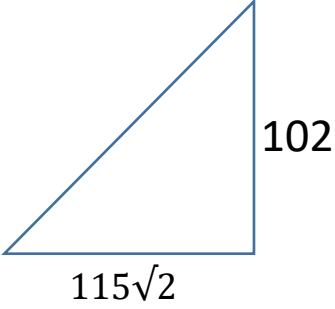
$$b = \frac{2\pi}{10} = \frac{\pi}{5}$$

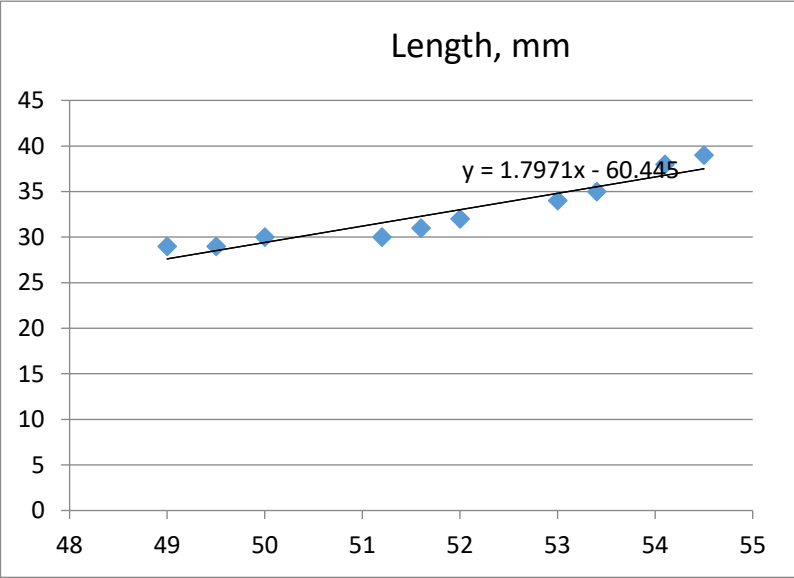
MS (0, 4, 8, 10)

LPC: Work of merit eg correct substitution of a coordinate into the sin equation

HPC: Finds either a or b correctly

Q8	Model Solution – 50 Marks	Marking Notes
(a)(i)		<p>MS (0, 5, 10)</p> <p>PC: At least 1 correct measurement</p>
(ii)	$\frac{a}{\sin a} = \frac{b}{\sin b}$ $\frac{a}{\sin 15} = \frac{100}{\sin 5} \quad (5^\circ = 180 - (160 + 15))$ $a = \frac{100(\sin 15)}{\sin 5}$ $a = 296.96169$  <p>SOH CAH TOA</p> $\sin(20) = \frac{h}{296.96169}$ $h = 101.5663 = 102m$	<p>MS(0,4,8,10)</p> <p>LPC: any work of merit in the right direction</p> <p>HPC: y found correctly and attempt to use this value to find h (or other correct method with just one error)</p> <p>** 9 marks for rounding and or units error**</p>
(b)	<p>SOH CAH TOA</p> $\cos 45 = \frac{x}{230}$ $x = 230(\cos 45)$ $x = 115\sqrt{2} \text{ m}$ <p>OR</p> $d^2 = 230^2 + 230^2$ $d = \sqrt{105800}$ $r = \frac{\sqrt{105800}}{2} \text{ m}$ <p>OR</p> $\cos 45 = \frac{230}{d}$ $d = 230\sqrt{2}$ $r = 115\sqrt{2} \text{ m}$	<p>MS (0, 4, 8, 10)</p> <p>LPC: any work of merit in the right direction</p> <p>HPC: any correct method with only one error in the calculation</p> <p>** 9 marks for unit error or surd equivalent to 162.63m **</p>

(c)	 $c^2 = a^2 + b^2$ $S_2 = (102)^2 + (115\sqrt{2})^2$ $S_2 = 36854$ $S_2 = 192$	<p>MS (0, 4, 8, 10)</p> <p>LPC: Pythagoras identified with some substitution</p> <p>HPC: 1 minor error in solution</p> <p>** 9 marks for rounding and or units error**</p>
(d)	$c^2 = a^2 + b^2$ $(192)^2 = (115)^2 + h^2$ $h^2 = 23639$ $h = 153.7$ <p>$S_2 = \text{Perpendicular height of the triangular faces}$</p> $\text{Area of the 4 faces} = 4 \left(\frac{1}{2}bh \right)$ $= 4 \times \frac{1}{2} \times 230 \times 153.7$ $= 70702m^2$ $= 71000m^2$	<p>MS (0, 4, 6, 8, 10)</p> <p>LPC: Work of merit to find perpendicular height</p> <p>MPC: Work of merit to find one area</p> <p>HPC: Multiplying the area of the triangle found by 4</p> <p>** 9 marks for rounding and or units error**</p>

Q9	Model Solution – 50 Marks	Marking Notes
(a)(i)	There is a weak/moderate positive relationship between the weights and lengths of the turtles. The longer the turtle, the heavier it will be.	MS (0, 3, 5) PC: Says it is a weak/moderate relationship or a positive relationship
(ii)	New Correlation Coefficient: 0.947453	MS (0, 4, 10) PC: Has an incorrect Correlation Coefficient within the range 0.94 and 0.955
(iii)	 <p>The figure is a scatter plot titled "Length, mm". The x-axis represents length in millimeters, ranging from 48 to 55 with major ticks every 1 unit. The y-axis represents weight in grams, ranging from 0 to 45 with major ticks every 5 units. There are 11 data points plotted as blue diamonds. A line of best fit is drawn through the points, with the equation $y = 1.7971x - 60.445$ displayed on the graph.</p>	MS (0, 3, 5) PC: Attempt at line of best fit
(iv)	Slope =1.8 Equation is $y = 1.7971x - 60.445$	MS (0, 3, 4, 5) LPC: Work of merit to find slope HPC: Gets a slope within the range of 1.5 -2.1 and working to find the equation of a line
(v)	Solution $y = 1.7971x - 60.445$ In this case $x = 60$ and we must find y $y = 1.7971(60) - 60.445$ $y = 47.381g$	MS (0, 3, 4 ,5) LPC: Identifies 60 as the input needed for the equation HPC: An answer in the range 40g – 55g (excluding the range for full marks) Full Credit: An answer in the range 45g -50g

<p>(b)</p>	<p>H0: The mean weight of the turtles in this area = 31g H1: The mean weight of the turtles in this area \neq 31g</p> $\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$ $32.7 - 1.96 \frac{2}{\sqrt{10}} < \mu < 32.7 + 1.96 \frac{2}{\sqrt{10}}$ $31.46g < \mu < 33.93$ <p>Reject H0: The turtles are heavier.</p> <p style="text-align: center;">OR</p> <p>Or $Z = \frac{x-u}{\frac{\sigma}{\sqrt{n}}} = \frac{32.7-31}{\frac{2}{\sqrt{10}}} = 2.69 \geq 1.96$</p> <p>The result is significant. Reject H0: The turtles are heavier.</p>	<p>MS (0, 4, 7, 8, 10) LPC: Some attempt at creating a confidence interval or finding a z-score but not completed</p> <p>MPC: Gets a Z-Score but doesn't use $\frac{\sigma}{\sqrt{n}}$</p> <p>HPC: Creating a confidence interval or found a z-score but a mistake is made or the work is not complete.</p> <p>Conclusion needed for full credit</p>
<p>(c)</p>	<p>H0: The mean length of the turtles in this area = 49mm H1: The mean length of the turtles in this area \neq 49mm Sample mean = 51.83</p> $Z = \frac{x-u}{\frac{\sigma}{\sqrt{n}}} = \frac{51.83-49}{\frac{3}{\sqrt{10}}} = 2.98 \geq 1.96$ <p>The result is significant.</p> <p>P-Value $2 \times P(Z > 2.98)$ $2 \times (1 - P(Z < 2.98))$ $2 \times (1 - 0.9986)$ $2 \times (0.0014) = 0.0028 < 0.05$ Therefore the result is significant</p> <p>We reject H0 and accept H1 The mean length of the turtles in this area \neq 49mm</p>	<p>MS (0, 4, 7, 8, 10)</p> <p>LPC: Some attempt at finding a z-score but not completed</p> <p>MPC: Gets a Z-Score</p> <p>HPC: Z score found correctly and correct conclusion, but P- Value not found correctly or the sentence "The mean length of the turtles in this area \neq 49mm" is not stated.</p>