

Q7	Model Solution – 60 Marks	Marking Notes
(a)		
(i)	$P = 80, P = y + 2x + y + \frac{2\pi x}{2}$ $80 = 2y + 2x + \pi x$ $2y = 80 - 2x - \pi x$ $y = \frac{80 - 2x - \pi x}{2}$	<p>MS (0, 4, 7, 8, 10)</p> <p>LPC: Forming an algebraic equation involving 80</p> <p>MPC: Any correct manipulation involving their equation</p> <p>HPC: Isolating the y term</p>
(ii)	$Area = 2xy + \frac{\pi x^2}{2}$ $Area = 2x\left(\frac{80 - 2x - \pi x}{2}\right) + \frac{\pi x^2}{2}$ $= 80x - 2x^2 - \pi x^2 + \frac{\pi x^2}{2}$ $= 80x - 2x^2 - \frac{\pi x^2}{2}$ $= 80x - \left(2 + \frac{\pi}{2}\right)x^2$	<p>MS (0,2,3,5)</p> <p>LPC: Forming an area equation</p> <p>HPC: Substituting y or their y into the area formula</p>
(iii)	$\frac{dA}{dx} = 80 - 2\left(2 + \frac{\pi}{2}\right)x$ $0 = 80 - 4x - \pi x$ $4x + \pi x = 80$ $x = \frac{80}{4+\pi} = 11.20$	<p>MS (0, 4, 7, 8, 10)</p> <p>LPC: Work of merit</p> <p>MPC: Correct differential or incorrect differential = 0</p> <p>HPC: Correct differential = 0</p>
(iv)	$\frac{d^2A}{dx^2} = -2\left(2 + \frac{\pi}{2}\right)$ $= -4 - \pi = -7.14$ <p><math>\therefore</math> maximum as answer is negative</p>	<p>MS (0, 6, 10)</p> <p>PC: Work of merit to find second differential</p>
(v)	$Area = 80x - \left(2 + \frac{\pi}{2}\right)x^2$ $= 80\left(\frac{80}{4+\pi}\right) - \left(2 + \frac{\pi}{2}\right)\left(\frac{80}{4+\pi}\right)^2$ $= 448\text{m}^2$	<p>MS (0, 3, 5)</p> <p>PC: <math>\therefore</math> Substitution of x</p> <p>**Incorrect units and/or rounding = 4 marks**</p>

<p>(b)</p> <p>(i)</p>	$V = \frac{1}{12}\pi h^2(3 - 4h)$ $\frac{dV}{dh} = \frac{2}{12}\pi h(3 - 4h) - 4\left(\frac{1}{12}\pi h^2\right)$ $= \frac{\pi h}{6}(3 - 4h) - \frac{\pi h^2}{3}$ $= \frac{\pi h}{2} - \frac{2\pi h^2}{3} - \frac{\pi h^2}{3}$ $= \frac{\pi h}{2} - \pi h^2$ $h = 0.1, \frac{dV}{dh} = 0.04\pi = \frac{\pi}{25}$ <p style="text-align: center;">OR</p> $V = \frac{1}{4}\pi h^2 - \frac{1}{3}\pi h^3$ $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$ $h = 0.1, \frac{dV}{dh} = 0.04\pi = \frac{\pi}{25}$	<p>MS (0, 4, 7, 8, 10)</p> <p>LPC: Any correct differentiating step OR substituting in 0.1 to incorrect differential</p> <p>MPC: Some correct use of product rule/correct differential with one mistake</p> <p>HPC: Correct differential/answer not in terms of <math>\pi</math></p>
<p>(ii)</p>	$\frac{dV}{dt} = \frac{\pi}{800}$ $\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt}$ $@h = 0.1, \frac{dh}{dt} = \frac{1}{0.04\pi} \times \frac{\pi}{800}$ $\frac{dh}{dt} = \frac{25}{\pi} \times \frac{\pi}{800} = \frac{1}{32} = 0.03125$	<p>MS (0, 4, 7, 8, 10)</p> <p>LPC: <math>\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt}</math> OR correct substitution into incorrect use of chain rule</p> <p>MPC: Correct <math>\frac{dV}{dt}</math></p> <p>HPC: Correct substitution into <math>\frac{dh}{dt}</math></p> <p>Units not needed for full credit</p>