**Question**

**(a)** If $z = 1 + i$ and $w = \frac{1}{z} + i$, find $w$ in the form $a + bi$.

$$w = \frac{1}{1+i} + i$$

$$w = \frac{1 + i + i^2}{1+i}$$

$$w = \frac{i}{1+i} \times \frac{1-i}{1-i}$$

$$w = \frac{i - i^2}{2}$$

$$w = \frac{-1+i}{2}$$

$$w = \frac{-1}{2} + \frac{i}{2}$$

**Scale 15 D [0, 5, 9, 12, 15]**

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|---|
| Low Partial: (5 marks) – Subs $z$ into $w$ |
| Mid partial: (9 marks) – attempt at writing $w$ as single fraction |
| High partial (12 marks) – Correctly expresses $w$ as single fraction |

**(b)** Show, using De Moivre's Theorem, that $cos3\theta = 4cos^3\theta - 3cos\theta$.

$$\cos 3\theta + \sin 3\theta = (\cos\theta + 1\sin\theta)^3$$

Expanding the RHS (NB this can be done using Binomial Theorem also)

$$\cos 3\theta + \sin 3\theta = (\cos\theta + \ i\sin\theta)(\cos\theta + i\sin\theta)^2$$

$$= (\cos\theta + \ i\sin\theta)(cos^2\theta + 2i\cos\theta\sin\theta + sin^2\theta)$$

$$= cos^3\theta + 2icos^2\theta\sin\theta - \cos\theta sin^2\theta + icos^2\theta\sin\theta - 2\cos\theta\, sin^2\theta - isin^3\theta$$

So, $\cos 3\theta + i\sin 3\theta = cos^3\theta - 3\cos\theta sin^2\theta + i(3cos^2\theta sin\theta - sin^3\theta)$

Equating Real parts gives....

$$\cos 3\theta = cos^3\theta - 3\cos\theta sin^2\theta$$

$$= cos^3\theta - 3\cos\theta(1 - cos^2\theta)$$

$$= cos^3\theta - 3\cos\theta + 3cos^3\theta$$

$$= 4cos^3\theta - 3\cos\theta \qquad \text{QED}$$

Scale 10C [0, 5, 7, 10]

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|---|
| Low Partial Credit: (5 marks) – Correct statement of De Moivre with 3 correctly inserted |
| Middle Partial Credit: (7 marks) – Arrives at equating coefficients correctly but fails to finish |