Algebra Question – Possible Solution

(a) Let $f(x) = x^3 + ax^2 + bx - 6$ where a and b are real numbers.
Given that $x - 1$ and $x - 2$ are factors of $f(x)$

(i) find the value of a and the value of b

$x - 1$ a factor $\Rightarrow x = 1$ a root

$f(1) = (1)^3 + a(1)^2 + b(1) - 6 = 0$

$f(1) = 1 + a + b - 6 = 0$

$a + b = 5$ ... (3m)

$x - 2$ a factor $\Rightarrow x = 2$ a root

$f(2) = (2)^3 + a(2)^2 + b(2) - 6 = 0$

$f(2) = 8 + 4a + 2b - 6 = 0$

$4a + 2b = -2$ ... (7m)

$2a + 2b = 10$

$2a = -12$

$a = -6$

$b = 11$ ...(10m)

(i) hence, find the values of x for which $f(x) = 0$.

I $(x-1)(x-2) = x^2 - 3x + 2$

$$x^2 - 3x + 2 \overline{)\, x^3 - 6x^2 + 11x - 6} \quad x - 3$$

...(3m)

$x^3 - 3x^2 + 2x$

$-3x^2 + 9x - 6$

$-3x^2 + 9x - 6$ ...(7m)

$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3 \Rightarrow$ Ans : 1, 2, 3 ...(10m)

(b) Find the solution set of $11 - 2n > 3$, $n \in \mathbb{N}$

I $-2n > 3 - 11$ ...(3m)

$-2n > -8$ ...(7m)

$2n < 8$

$n < 4$

$\Rightarrow \{0,1,2,3\}$ ...(10m)