CONTENTS

ACKNOWLEDGEMENT

The IMTA would like to thank The Celtic Press for sponsoring the cost of producing this Newsletter.

EDITORIAL

This edition of the Newsletter marks a new beginning as it is the first edition for your new editor. I ask your forbearance in advance for the inevitable hitches and glitches which fall the way of every publication. My admiration goes to those editors who have preceded me for the many valuable publications which have marked the years of the Irish Mathematics Teachers' Association. I extend to them the thanks of all members.

A Mathematics Newsletter can take many formats and fulfill many functions. Contained in this edition are some of the ideas which may be explored in future. The survey of members interest in topics shows a valuable list of possibilities. High up on the list of interests there is the convenient printed version of Answers to Leaving Cert Question Papers. In this issue you will find the Solutions to the Leaving Certificate Higher Mathematics Papers 2005.

At present there is the opportunity to become involved in the grand debate which will go to the heart of maths teaching. The NCCA have opened the discussion with their document *Review of Mathematics in Post-Primary Education*. This is accessible at http://www.ncca.ie/ . The Newsletter should both record and advance the discussion involved. In their Consultation Questionnaire (which was available to teachers and non-teachers alike — closing date 16th December, 2005, now past) the NCCA have set out a comprehensive range of headings to initiate the discussion. The Questionnaire invited comments on ideas such as *The Role and Purpose of mathematics education* as well as *Concerns regarding mathematics*, *Mathematics in relation to other subjects, The Influence of the examination papers, Teaching and learning in mathematics* among others.

Methods of topic introduction and development may be presented in the forum of this Newsletter also. The NCCA has published an introductory *Guidelines for Teaches for the Junior Certificate*. This is a comprehensive and valuable document. It shows, for example, a number of ways of approaching Pythagoras' Theorem through drawing, construction of artifacts and calculation. In this Newsletter there is a further contribution to the exploration of Pythagoras' Theorem through drawing showing that each topic has a vast number of entry points and development paths. Furthermore, at inservice days around the country a number of tutors are strenuously bringing the message of active methodologies in mathematics teaching to a widening audience. Ideas for active methodologies are particularly welcome to this Newsletter.

At another level there is the question-point that if all teachers improved their teaching methods across the country would it become apparent in the appearance of improved grades in the public examinations? A serious debate on this point is overdue. The quantitative analysis may require expansion to some qualitative analysis. What do teachers think about the course that they are teaching? Is there any value in knowing something of the history of mathematics and the place it has played in the cultural development of the world? Where is the space to present these ideas to students? How do our pupils view our attempts to move outside the examined syllabus - as wasted distractions or as valuable expansions of appreciation and education?

The past year contained a number of significant historical reference points. First of all, there was the celebration of the 40th anniversary of the founding of the Cumann Oidí Matamaitice Na hÉireann/Irish Mathematics Teachers' Association (in 1964). Secondly, at a National level it is significant that the year of 2005 was designated as Hamilton Year in order to commemorate the genius of Sir William Rowan Hamilton. While many events have taken place to mark that commemoration it seems appropriate to devote some extra space in the Newsletter - even as a retrospective view following the year 2005. This will take place in a future edition.

As you page through this edition of the Newsletter hopefully you will be inspired. enthralled and intrigued by the various contributions. Enjoy a good read. With heartfelt thanks to all contributors.

In the next issue there will be Solutions to Leaving Certificate Higher Applied Mathematics 2005 and, just to whet your appetite for elections, an article on the mathematics associated with election counts.

All contributions are welcome.
Members may wish to invite submissions for publication from others who are doing research or simply have a general interest in mathematics.

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Neil Hallinan

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Cumann Oidí Matamaitice na h-Éireann
Irish Mathematics Teachers' Association

The IMTA was founded in 1964 to promote and assist the teaching of mathematics at all levels.

Membership is open to all those interested in mathematics and mathematics education.

As the IMTA is represented on all NCCA mathematics course committees, members, through their Branch meetings and syllabus committee representatives, have a direct input into syllabus revisions and curriculum changes.

Membership may be obtained through a Branch organisation. Fee: €12.70.

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SURFIN' FOR MATHS

http://www.teachnet.ie/tbrophy/concepts.html (Update: link inactive - Ed.)
This site introduces Geometry through use of visualisations and interactions. The Plane is shown as an ever-expanding surface; Lines and Half-lines are shown with flying birds;similarly, Line Segments and Collinear Points are shown with tumbling and falling birds.

http://www.people.memphis.edu/~brveteto/ (Update: link inactive - Ed.)
Set up by Bette Veteto as a basis for report-writing for Maths classes on a number of topics. This site also contains a list of reference sites.

http://www.cut-the-knot.org/index.shtml
Interactive pages on mathematics by Alexander Bogomolny.

If you know of other sites which you have found useful and wish to share please e-mail

to hallinann@eircom.net for inclusion in the next edition of the Newsletter.

A SURVEY OF MEMBERS INTEREST IN TOPICS FOR THE NEWSLETTER

What would you like to see in the Maths Newsletter?

1. National Developments – work by the NCCA
2. Irish Junior Maths Competition – from the Cork Branch
3. Team Math Competition
4. Solutions to Higher Level Maths Papers
5. Maths Olympiad Competition
6. Rainy Day puzzles
7. Reports from Branches
8. Maths and the Internet
9. History of Mathematics
10. Careers in Mathematics
11. Junior Cert Maths Support Service
12. Developing technology – the calculator and beyond
13. How to Start teaching a Topic in the Classroom
14. Subject Convenors reports
15. International news – ICME (International Congress of Math Education)
16. Research in Mathematics
17. Non-specific items of interest
18. Contributions from Universities
19. Maths and computers
20. Book Reviews
21. Reports from the Maths Council
22. Solutions to Higher Level Applied Maths Papers
23. Maths in other subject areas
24. National Science Week Maths Competition – from NUI Maynooth
25. Issues of concern
26. Other (specify):

This list of topics is ranked according to the results of the survey taken on 26th/27th November 2005 at the IMTA AGM in Tullamore, Co. Offaly.

Should you wish to add to this list your comments or contributions will be most welcome.

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Fr. Ingram Memorial Lecture

Second Level Maths Education in Ireland 2005 and beyond

"Maths Questions"

Peter Tiernan — Tullamore 2005

In this paper I propose to outline the present position of Maths at second level and to describe some of the possible ways forward for Maths Education in Ireland. While this is not a research paper I have used other people's research extensively in writing the paper; all of whom I acknowledge and thank.

Maths is seen by many people as a necessary evil --- necessary because you cannot get a good job or high points in the LC without it and evil because it because it uses a language and symbols that are accessible only by the chosen few. The public attitude to Maths was very well articulated by Rob Eastaway at a recent lecture when he gave 10 ways to recover at a party after you have told someone you earn your living teaching maths.

The following is an extract from Hansard:

> **Mr.McWalter MP**……..*I put this on the agenda because I have been troubled since the President of a teachers union suggested that maths might be dropped as a compulsory subject at the age of 14.He cited the quadratic equation as the sort of irrelevant topic that pupils study I had hoped that the government would make a robust rebuttal, but their was no defence of maths in general or of the quadratic equation in particular.*
>
> *If such assertions are left unrebutted, what was an ignorant suggestion at one time can become received wisdom a very short time later and an article of educational faith a very short time after that.*
>
> *He was happy to teach maths to those who enjoyed it, but he wanted to stop teaching it to those who did not. By defending the centrality of the quadratic equation to maths education I hope to submit some thoughts on what we would be missing if we allowed maths to be regarded as a subject of no greater worth than any other subject.*
>
> *Those who tell us that we need no familiarity with quadratic equations are telling us to ignore 400 years of technological intellectual and scientific development. When educators tell us to do that I rejoin they have a very strange view of education which I would like the Government to repudiate.*
>
> *It is powerful educational medicine to come to understand that what can be expressed very simply can be extraordinarily difficult to solve. The quadratic equation can teach us to be humble.*

The notion that a statement, if not rebutted, can quickly become an article of faith applies to the situation Maths finds itself in now --- the argument that it is for the few should not be accepted.
The review of Maths being carried out by the NCCA was prompted by a number of factors

(i) Maths experience at 2nd level for all partners (students, teachers, parents etc)
(ii) Chief Examiner's report
(iii) PISA
(iv) Uptake at Higher Level and at Foundation Level.
(v) Performance at 3rd level

We might add in the high number taking private tuition, some as early as first year. A recent report presented by Judith Ireson found that Maths is the subject worldwide that students are most likely to go for extra tuition.

The evidence from these and other sources suggests that we are doing many things extremely well (Olympiad, Team Maths, gender balance, and the work done by the IMTA). On the other hand it also suggests that we need to look at:

(i) The purpose of the Maths education we now offer
(ii) What we are teaching (does it suit the student at his/her stage of development)
(iii) The way we are teaching it (training and CPD of maths teachers)
(iv) The way we are assessing it

Purpose of Maths Ed

.....an individual's capacity to identify and understand the role that Maths plays in the world, to make well-founded Maths judgements and to engage in maths in ways that meet that individual's current and future life as a constructive concerned and reflective citizen (PISA)

.... Recognises the importance of Maths for it's own sake as an intellectual discipline, for the knowledge economy, for Science Technology and engineering, for the work place and for the individual citizen (Sir Adrian Smith)

... problem solving, reasoning and proving, reflecting, selecting tools and strategy, connecting, representing, communication of results (Canada)

We can see from the above that there is not universal agreement on a statement of purpose for Maths Education. I suggest that this is an issue we need to address: interested parties should initiate research and thereby facilitate enlightened discussion in their communities and then bring forward their definitions so that an informed debate can take place. If we agree on the central role Maths has in a sophisticated and civilised society then we owe it to the coming generations to perform this exercise honestly and expeditiously.

THE QUESTION*: WHAT IS OUR PURPOSE?*

What we are teaching

Our courses are dominated by Algebra and Geometry and to judge by the Chief Examiner's report many students fail to grasp even the fundamental skills of both of these topics. A report in 1999 said little had changed in this regard since the 1996 and the same will be said in 2005. Why?

The problem referred to in the next quotation refers to the situation at Leaving Cert Ordinary level where a large section of the cohort following this course and sitting the examination is not suited to either.

It is clear, both from the continuing relatively high failure rate and from the type of work presented by the candidates who are failing, that there are significant numbers of candidates who are wholly unsuited to taking this exam. It is difficult to see what purpose is served by students following a course that quite evidently is not meeting their educational needs. The quality of the mathematical learning experiences such students will have on leaving the system is not satisfactory.
Chief Examiners Report 2001 ATGL

Are we failing to take into account the fact that students of the same age may not be achieving at the same level at the same time? Do we see '*Maths as a ladder the rungs of which are the various levels which have to be climbed as quickly as possible or as a series of hurdles over which a student must pass before moving on to the next one (NZ)'*

It might help if when formulating a syllabus the judgement of experienced teachers as to what students can do at various stages could be combined with up to date research into Math learning to place material in appropriate levels. The reformatting work being carried out on the JC takes some of this into account. The provision of Aims and Objectives for each topic should be helpful to teachers and students. If the syllabus is set out in terms of achievement objectives then the achievement objective can be realised at a range of standards.

PISA suggested that we were looking after most of the students quite well but maybe not doing enough for the better student. In this regard Smith suggests

...an extension curriculum and assessment framework firmly rooted in the current Programme of Study but pupils should be presented with greater challenges ..... harder problem solving in non standard situations, a greater understanding of Maths interconnectedness and a greater facility in reasoning including proof and an ability to engage in multistep reasoning and more open-ended problem solving.

followed by

> *Up to 3 Curr and Assess studies of variants ……including trialling, feedback and modification and an assessment of workload implications.*

THE QUESTION*: ARE OUR STUDENTS AT THEIR APPROPRIATE LEVEL?*
The way we teach it

A report on Maths Ed in the US ---- ***Before it's too late*** --- says that the approach to teaching Maths had changed little in two generations.

(i) Review + homework
(ii) Example
(iii) Drill on low level procedures
(v) Exercise
(vi) Check
(vii) Homework

Is it familiar?

Inside Classrooms ---- Seems to indicate that for many teachers here little has changed in their presentation.

We should remember the following

> *…… teachers should not assume that because students can work through a set of similar closed form exercises successfully that they will then have sufficient understanding to identify contexts where that knowledge is required, apply the mathematical tools appropriately and interpret the results correctly. (Ch 2001)*

The JCMSS was in favour of a hands-on approach to support a "teaching for understanding" method in the classroom. This initiative has not yet been evaluated but the anecdotal evidence would suggest that those who gave up their old ways are happy with the results.

> *Could the performance of students be enhanced with learning experiences in which conceptual understanding is more strongly emphasised and relationships between math knowledge and real life applications are more obvious. PISA*

An interesting alternative approach to teaching Maths is outlined in both international reports.

In Japan closely supervised collaborative work by students is the norm. Teachers begin a class by setting a problem employing principles they have not yet learned. Students work alone /small groups. They are then called to present their work and the whole class works through the problem to uncover the related Maths concepts. The students learn through reasoned discovery not through lecture alone. This style of teaching has of course come from a different learning and teacher training background.
Lesson planning 2 – 5 hours per week.

THE QUESTIONS: *DO WE NEED TO LOOK AT OUR PRESENTATION? IS IT ENOUGH TO SAY IT WORKED FOR ME 30 YEARS AGO?*
Assessment

Traditionally we have gone for the mark/grade form of assessment, with little or no diagnostic element. The Maths Review Commission in NZ reported

> *……. Diagnostic procedures which enable teachers to discover individual difficulties children may be having should be tried. Appropriate diagnostic testing may reveal that the reason for a particular student's lack of progress is a gap in understanding of a skill taught at an earlier stage. Such a gap may be easily filled once identified.*

Examples from our own experience would seem to verify this.

Assessment should focus on what a student knows and should provide students and parents with an indication of a student's progress. Teachers could comment on what the student has been working on, what they have achieved and how they have achieved it.

THE QUESTION: *SHOULD WE TRY A DIAGNOSTIC STYLE OF CLASSROOM ASSESSMENT?*
The way we assess at the public exams

Our system with reference to slips and blunders.

NSW system; Syllabus layout:
(i) No choice
(ii) Marks on Question Paper
(iii) Marking Criteria
(iv) Mapping Grid
Relevant documents are appended.

THE QUESTION: *DO WE NEED TO REFRESH OUR MARKING SYSTEM?*

THE QUESTION: *WOULD YOU ACCEPT FOR **ATAL:***
PAPER 1 CORE PAPER: NO CHOICE 6 FROM 6
PAPER 2 HARDER QUESTIONS: (A) 5 DO 5 + (B) ONE "DEAL BREAKER" WHERE A PROBLEM-SOLVING QUESTION IS POSED AND A SPECIFIED STANDARD MUST BE ACHIEVED IN ORDER TO GET A1?

References

(i) *Review of Post Primary Maths …… NCCA*

(ii) *Before it's too late…… NCMST*

(iii) *Chief Examiner's Report …… DES*

(iv) *PISA report…… ERC*

(v) *Maths in NZ curriculum…… Ministry of ed NZ*

(vi) *Ontario Curriculum…… Ministry of Ed*

(vii) *Making Maths Count …… DES (UK)*

(viii) *International Trends in PP maths ed……..NCCA*

(ix) *101 Uses of a Quadratic Equation……Chris Budd/Chris Sangwin*

Some Innovative Projects to Watch Out For

The Numbers Game: Game-based eLearning for Numeracy and Basic Mathematical Skills delivered via web and mobile phone (from The Cork Institute of Technology).

Engineers Ireland: **Steps to Engineering—** a programme for schools aimed at tempting students into science, maths and engineering careers. www.steps.ie

Science Week Ireland organised by **Discover Science and Engineering:** www.discover.ie

The Infinity Project: Applying fundamental maths and science to modern digital technology. The Dublin Institute of Technology (DIT) have used the Infinity Project as an integral part of their First Year Programme in the School of Electronics and Communications Engineering at Kevin St., Dublin. The Head of Learning and Teaching Innovation, Michael Tully, hopes to extend the programme to Secondary Schools.
http://www.electronics.dit.ie/

USING TECHNOLOGY TO TEACH MATHEMATICS

Introduction:

Technology has progressed to extend the human person: much as a spider's web extends the spider. A great teaching tool that has emerged in eight schools in the country this year is an interactive whiteboard. It is a system consisting of a single computer, whiteboard, projector and printer. The projector displays the image from the computer onto the whiteboard. BECTA research in the United Kingdom has shown that the exam results exceedingly improved when introduced to secondary schools in the UK.

Teaching Maths using the Technology:

I have found the teaching of maths greatly enhanced by such a facility. Included are resources such as mathematical templates making graphing functions (see fig. 1) effortless for the

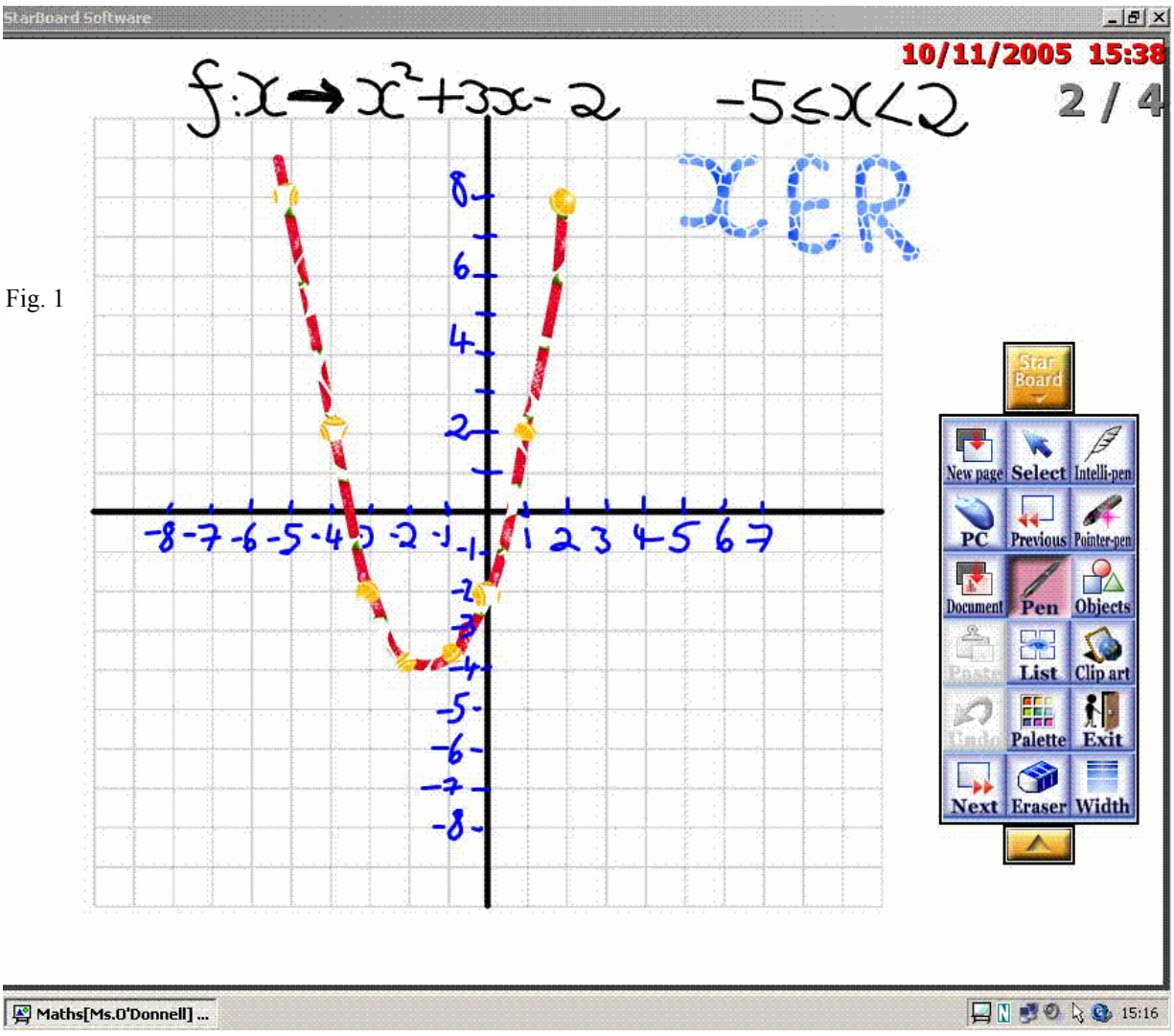

Fig. 1

teacher. The scales of graphs are more accurate when drawn using a mathematical template. Accuracy as a mathematical skill is very significant. A palette of colours provides vivacity and richness to the graph.

The Value of the Toolbar:

There are many useful icons, on the toolbar, which are shown, in fig.1 to the right of the graph drawing.

- The drawing of circles and perfect shapes was never easier, accomplished through using the 'intelligent pen' or selecting the objects icon.

- The 'list' function proves to be very beneficial in class. At the end of a class, it is possible to review all the maths work, accomplished on the interactive whiteboard. All the teacher has to do is click the 'list' function to display the class work.

- When using a blackboard, once the wiping off material occurs, that information permanently deletes, unless the teacher chooses to rewrite it again. With the interactive whiteboard, every new page saves in a list. At the end of the class, the teacher has the option to save, discard or delete the data. If the teacher chooses to save the data, then the data is retrievable next class. If a student happens to be absent for a particular class, the saved data can be printed.

- Students since September are eliciting great enthusiasm. They are eager to go up to the board at every available opportunity!

- Internet can be accessible in every maths class when using the interactive whiteboard. The teacher can type in the site on the computer with the result that it comes up on the board. I have used the following website to show the students information on William Rowan Hamilton since it is the bicentenary of his birth.

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Hamilton.html

Conclusion:

The development of mathematical skills impinges on development opportunities for the individual. There are ensuing economic implications in a society increasingly reliant on, and influenced by, advances in science and technology which have a high dependency on mathematical principles. Through the school's involvement in the interactive whiteboard project, I feel the school is endeavouring further to inculcate students with a sense of the beauty and joy of a subject such as mathematics.

Mary O'Donnell
Dublin

DRAWING ON PYTHAGORAS

Project

To illustrate Pythagoras Theorem by drawing on squared paper

Resources:

Squared paper (from a Sum Copy)
Pencil
Ruler
Calculator

Strategy:

- Place four congruent right-angled triangles 'back-to-back' in a 'St. Brigid's Cross' formation. By construction the adjoining angles sum to 90º so that the shape is a square with the hypotenuse of the triangle as the length of its side. If the other two sides are unequal then a small square is also formed in the centre.

- Attach a further congruent triangle on one edge. Find the area of the large square now formed. (We are now looking at the square on the hypotenuse of a right-angled triangle).

- Form the squares on the other two sides of the triangle and find the separate areas of both.

- Notice that the total area of the two smaller squares is equal to the single area of the large square.

Stage 1: Draw a St. Brigid's Cross shape (Figure 1, 2). Complete the square by joining the end points (Figure 3).

A convenient method of drawing the square on the hypotenuse is to start at a 'centre' point and draw continuously using the lengths of the sides of the triangle following the arrows as illustrated.

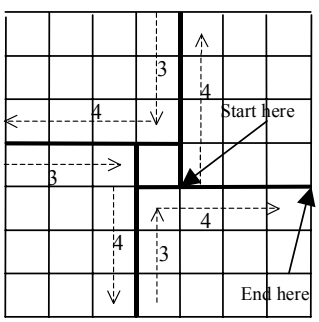

Figure 1

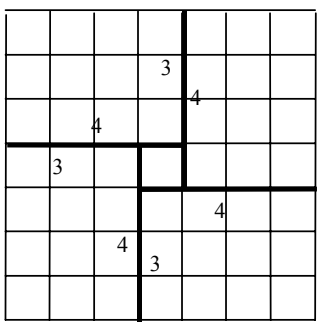

Figure 2

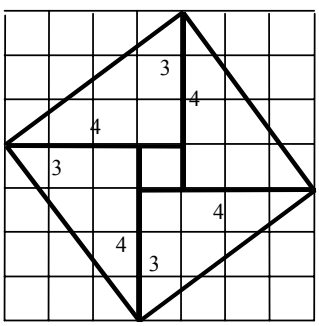

Figure 3

Stage 2: Attach the fifth triangle (Figure 4) with its hypotenuse against the side of the square. This allows an easy method of finding the area of the square on the hypotenuse: a rectangle has been formed whose area may be counted in small square units of the sum-copy; each triangle is half of this number of small squares; there are four such triangles in the large square; finally, add on the area of the middle square.

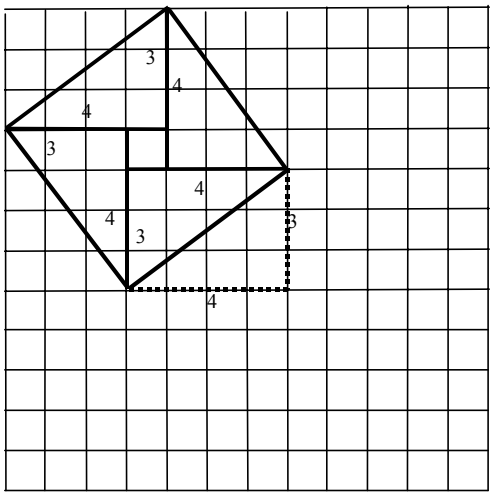

Figure 4

The area of the rectangle is 4 x 3 = 12

The area of a triangle is 12 ¸ 2 = 6

The area of four triangles is 4 x 6 = 24

The area of the middle square is 1

The area of the square on the hypotenuse of the triangle is 24 + 1 = 25 ie. 25 units of the Sum-copy squares.

Stage 3: Draw squares on each of the other sides of the fifth triangle (Figure 5) and find their area.

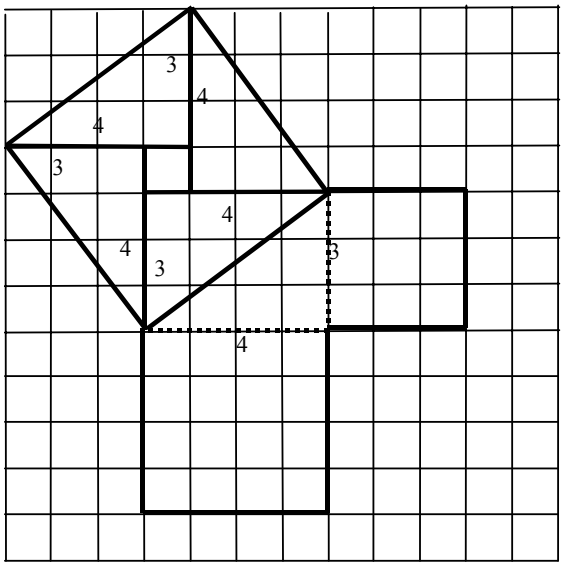

Figure 5

The area of the square on the side of the triangle with length 3 is 3 x 3 = 9

The area of the square on the side of the triangle with length 4 is 4 x 4 = 16

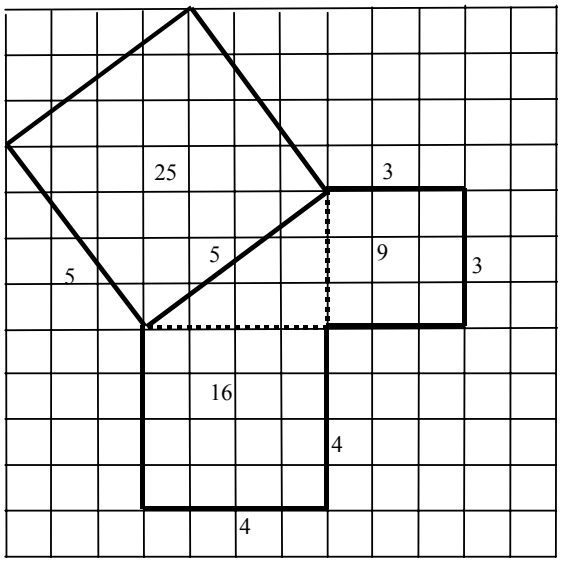

Figure 6

Stage 4: Notice the connection which appears between the areas of the squares: the areas of the two smaller squares total to give the area of the single larger square (Figure 6).

Pythagoras Theorem

Area 9 + Area 16 = Area 25

The area of the square on the hypotenuse of a right-angled triangle
is equal to
the sum of the areas of the squares on the other two sides of the triangle

Development 1: The exact length of the side of the large square can now be found without using a ruler if we wish.
The side length must be a number which divides the same number of times as itself into 25: ie. the answer is the square root of 25 which is 5
We can write $\sqrt{25} = 5$ is the length of the hypotenuse of the triangle.

Development 2:
This method may be used with any two lengths of sides of a right-angled triangle.
Notice that in cases where two sides of the same length are used there is no 'middle square'.

Development 3:
This method may be developed into a proof of Pythagoras Theorem.

In a right-angled triangle let the lengths of sides be a, b and hypotenuse length c,with a ³ b.
We wish to show that $a^2 + b^2 = c^2$

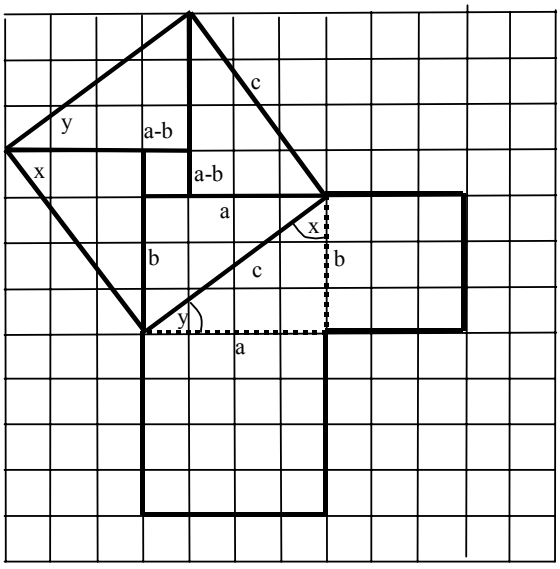

Figure 7

Construct the square on the hypotenuse as shown (Figure 7). Length a-b ³ 0.

The area of the large square is c^2.

The area of the rectangle is a x b (or ab); the area of the triangle is ½(ab).

By construction the area of the large square also is
4(ab),2 + (a-b)²
(area of 4 triangles and the 'middle square').

i.e. $2ab + a^2 - 2ab + b^2$ But this is the same as c^2
\ $2ab + a^2 - 2ab + b^2 = c^2$
\ $a^2 + b^2 = c^2$.

Note: |angle x| + |angle y| = 90^o .

Development 4: Find out more about Pythagoras by visiting some sites on the Internet.
http://www.mathsyear2000.org/timeline/ (Update: link inactive - Ed.)
This site gives a thumb-nail sketch of many mathematicians along a time-line.
http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html
This site gives a more extensive history of Pythagoras.

Neil Hallinan
Dublin

SONGS WITH NUMBERS IN THEIR TITLE

How many songs do you know which have 'number words' in their title?
And who sang them? Here is a quick reference guide:

	Song Title	Artist
0.	Down to Zero	Joan Armatrading
1.	One	U2
2.	Two More Bottles of Wine	Emmylou Harris
3.	Three Little Birds	Bob Marley
4.	Four Strong Winds	Neil Young
5.	5D	The Byrds
6.	From a Buick 6	Bob Dylan
7.	Seven Spanish Angels	Ray Charles and Willie Nelson
8.	Eight Days a Week	The Beatles
9.	The One After 9.09	The Beatles
10.	Perfect 10	Beautiful South
11.	7 – 11	The Ramones
12.	Twelfth of Never	Nat King Cole
13.	Thirteen	Big Star
14.	14 Years	Guns n' Roses
15.	5.15	The Who
16.	Sixteen Tons	Tennessee Ernie Ford
17.	At Seventeen	Janis Ian
18.	Fly 3.18	The Jam
19.	19	Paul Hardcastle
20.	20-20 Vision	Rory Gallagher
21.	21 Questions	50 Cent
22.	22 Acacia Avenue	Iron Maiden
23.	22 Going on 23	Butthole Surfers
24.	Twenty-four Hours from Tulsa	Gene Pitney
25.	Twenty-five Minutes to Go	Johnny Cash
26.	26 Miles (Santa Catalina)	The Four Prefs
27.	27 Forever	A Certain Ratio
28.*	No Clause 28	Boy George
29.	No. 29	Steve Earle
30.	30 Days	Chuck Berry
31.*	31 Flavors	King George
32.	32 –20	The Flamin' Groovies
33.	The Pilgrim: Chapter 33	Kris Kristofferson
34.	#34	Dave Matthews Band
35.	Rainy Day Woman #12 and 35	Bob Dylan
36.	36 Inches High	Nick Lowe
37.	We Do What We're Tole (Milgram's 37)	Peter Gabriel
38.*	38 Years Old	The Tragically Hip

	Song Title	Artist
39.	'39	Queen
40.	Forty Shades of Green	Johnny Cash
41.	American Skin (41 Shades)	Bruce Springstein
42.	42nd Street	Musical: 42nd Street
43.	Section 43	Country Joe and The Fish
44.	Care of Cell 44	The Zombies
45.	5.45	Gang of Four
46.	Forty Six and 2	Tool
47.	Texas 1947	Guy Clark
48.	48 Hours	The Clash
49.	Highway 49	Mose Allison
50.	Fifty Ways to Leave Your Lover	Paul Simon
51.	Highway 51 Blues	Bob Dylan
52.	52 Girls	The B-52's
53.	53rd and 3rd	The Ramones
54.	Car 54, Where Are You?	TV sitcom theme song
55.	Ol' 55	Tom Waits
56.*	Along for the Ride ('56 T-Bird)	John Denver
57.	57 Channels and Nothin' On	Bruce Springstein
58.	Poem 58	Chicago Transit Authority
59.	59th Street Bridge Song (Feelin' Groovy)	Simon and Garfunkel
60.	Sixty Sixty	Faust
61.	Highway 61 Revisited	Bob Dylan
62.	Rocket Reducer No. 62	The M C 5
63.	December, '63 (Oh, What a Night!)	The Four Seasons featuring Frankie Valli
64.	When I'm Sixty Four	The Beatles
65.	65 Love Affair	Paul Davis
66.	(Get Your Kicks On) Route 66	Nat King Cole
67.	Back in '67	The Average White Band
68.	Questions 67 and 68	Chicago Transit Authority
69.	Summer of '69	Bryan Adams
70.	Casanova 70	Air
71.*	Kentucky Feb. 27, '71	Tom T. Hall
72.	72	Turin Brakes
73.*	Psalm 73 (My God's Enough)	Barlow Girl
74.	74 – 75	The Connells
75.	74 – 75	The Connells
76.	Seventy-six Trombones	Musical: The Music Man by M. Willson
77.	Seventy-seven	Sunset Street
78.	78 Stone Wobble	Gomez
79.	Winter of '79	Tom Robinson
80.	80	Green Day
81.*	Stay ('81)	Madonna
82.*	No. 82	Bonnie Owens
83.*	'83 Medley	John Magee
84.*	When I'm 84	The Beautiful South
85.	Nineteen and Eighty-five	Paul McCartney and Wings
86.	86	Green Day

	Song Title	Artist
87.*	'87 And Cry	David Bowie
88.	88 Lines about 44 Women	The Nails
89.	Pop Song 89	R.E.M.
90.	Blimps Go Ninety	Guided by Voices
91.*	Economics '91	Ill Bill
92.*	92 Degrees	Souxie and the Banshees
93.	93 Million Miles	30 Seconds to Mars
94.*	Prayer '94	Bon Jovi
95.*	Total Hate 95	Gwen Stefani
96.	96 Quite Bitter Beings	CKY
97.	'97 Bonnie and Clyde	Eminem
98.	98.6	Keith
99.	99 Red Balloons	Nina
100.	One Hundred Tears	The Cure

Colette Nolan and friends. With Internet help on the starred entries!
Dublin

R.W. C.

There is **an identity** which links the number of primes, twin primes and twin non-primes*.

This identity is based on the number of pairs of the form (6k-1, 6k+1) which exist within the Natural numbers.

Thus

The **number of pairs** of the form (6k-1, 6k+1) *(which is 'k' for n of the form 6k+1)*
Equals The number of twin non-prime pairs *(which is $\pi_o(n)$)*
Plus The number of primes *(which is $\pi(n)$)*
Minus The number of twin primes *(which is $\pi_2(n)$)*
Minus 1

$k = \pi_o(n) + \pi(n) - \pi_2(n) - 1$ for *n* of the form 6k+1, k a Natural number > 0.

e.g., For n = 13 (ie. 6(2) + 1); k = 2; Number of twin non-primes = 0;
Number of primes = 6 (these are 2,3,5,7,11,13);
Number of twin-primes = 3 (these are (3,5), (5,7), (11,13)).

$2 = 0 + 6 - 3 - 1$

Check it out: Try the formula for n = 121

*Note: k= 20 and $\pi_o(n) = 1$ which is the pair (119,121). This is the first such pair of non-prime numbers of the form (6k-1, 6k+1) - a twin non-prime!

Now you need to find $\pi(n)$ and $\pi_2(n)$ the number of primes and the number of twin prime pairs up to 121.

Finally, check that $20 = \pi_o(121) + \pi(121) - \pi_2(121) - 1$.

A challenge: Find the next twin non-prime.

Neil Hallinan

NUI Maynooth Science Week On-line Quiz

SENIOR SECTION

1. What Greek philosopher created the famous paradox about Achilles and the Tortoise?

2. The word Cartesian is derived from which French mathematician's name?

3. In what Meath town did William Rowan Hamilton spend his early years?

4. Name the Scottish mathematician who created logarithms.

5. Two married couples and their single friend went shopping. Together they spent 100 euro on 10 items. No one bought nothing and no one bought more than 4 items. Each item cost a whole number of euros.
Dave bought his wife a dress but nothing for himself, in total he and his wife spent 65 euro. One woman spent 50 euro and a man spent 7 euro. The person who bought the card also bought the keyring and the wallet (which cost the same as each other). She did not spend the most. Clare is married to Eoin. Four people each bought themselves a doughnut to eat. One woman was pleased to receive flowers from her husband even if they were in a sale at 5 euro. Brenda bought the briefcase. Ann spent the least as she is saving money for her wedding. The card cost 3 times as much as a doughnut. Who is maried to who, who bought what and how much did it cost?

6. The numbers 1,2,3, ... , 2005 are written on a blackboard. You can rub out any two of them and replace them with their difference: for example, you can replace 5 and 13 by 8 (or -8). Can you do this in such a way that eventually you are left with only zeroes on the blackboard and no other numbers?

7. A diagonal is drawn through a grid of square boxes. The grid is 2005 boxes long and 1066 boxes high. How many boxes does the diagonal cut through?

8. Show that the polynomial $x^5 + 2x^4 + 9x^3 + 6x^2 + 6x + 4$ has no rational roots.

9. A rockfal traps a scuba-diving class in an isolated cave by the sea. There is only one way out which involves swimming through a long underwater passage. The class has only one oxygen tank and either one person can use it or two people can share it on each trip. The instructor, I, can swim the passage in three minutes but the three students, A, B and C, take 6, 15, and 30 minutes, respectively. If two people travel simultaneously, the pair must go at the speed of the slower swimmer. The incoming tide will completely flood the cave in an hour and it would take longer than that to get help, so the only options involve several trips back and forth through the passage until everyone is out. Unfortunately no trip is possible without the oxygen tank, and the tank contains only enough oxygen for 51 minutes of underwater travel. How should the trips be organised to get everyone out safely?

10. Let $P_1, P_2, \ldots P_n$ be points on a sphere of radius 1 in $\mathbb{R}^3$. Let d_{ij} be the distance in $\mathbb{R}^3$ from P_i to P_j. Prove that $\sum_{i<j} d_{ij}^2 \leq n^2$.

NUI Maynooth Science Week On-line Quiz

JUNIOR SECTION

1. On what island was Pythagoras born?

2. What animal is involved in the original creation of the Fibonacci sequence?

3. In which decade was Fermat's Last Theorem proved to be true?

4. Which Greek mathematician shouted 'Eureka' after a famous discovery in the bath?

5. What is the least number of coins you need in order to buy any item costing 1 euro or less with exact change?

6. There are 24 kilos of rice in a sack. Can you measure out 9 kilos using only a balance?

7. Cross out ten digits from the number 123451234512345123451234512345 so that the remaining number is as large as possible.

8. The four digit integer $aabb$ is a perfect square. What are the values of a and b?

9. Suppose there are 2005 people in a room. Is it possible that each is related to exactly three other people in the room?

10. Prove that $x^4 + y^4 + 18 \geq 12xy$ for all real numbers x and y. Determine also when both sides of the inequality are actually equal.

Notice: A list of winners and links to Solutions are provided on the NUIM web-site at
http://www.maths.nuim.ie/staff/sbuckley/contest/rules05.html

SENIOR SECTION SOLUTIONS

1. Zeno.

2. Rene Descartes.

3. Trim.

4. John Napier.

5. Brenda is married to Dave, Clare is married to Eoin. Ann bought a doughnut (2 euro). Brenda bought a doughnut (2 euro) and a briefcase (48 euro). Clare bought a doughnut (2 euro), a keyring (9 euro), a wallet (9 euro) and a cord (6 euro). Dave bought a dress (15 euro). Eoin bought a doughnut (2 euro) and flowers (5 euro).

6. No. Since $x + y$ and $x - y$ always have the same parity, the parity of the sum of numbers on the board is preserved throughout the process. The sum of the first 2005 numbers is 2011015, which is odd. If the numbers on the board could be reduced to zeros, the parity would have to be even.

7. 3070. The required number of boxes is one less than the sum of the number of rows and the number of columns, provided the number of rows and the number of columns have no common factors (which is the case for 2005 and 1066).

8. Assume $\frac{p}{q}$ is in its lowest terms, so $\gcd(p, q) = 1$. Replace x in the polynomial by $\frac{p}{q}$ and set this expression equal to zero. Multiply across by q^5 and rearrange to make p^5 the subject. This gives

$$p^5 = -q(2p^4 + 9p^3q + 6p^2q^2 + 6pq^3 + 4q^4).$$

So q divides p^5, but $\gcd(p, q) = 1$, so q must be 1 or -1. Also

$$p(p^4 + 2p^3q + 9p^2q^2 + 6pq^3 + 6q^4) = -4q^5.$$

Therefore p divides 4, so p must be $1, 2$ or 4. Thus $\frac{p}{q}$ must be $1, -1, 2, -2, 4$ or -4. Trying these values in the polynomial shows that none are roots.

9. I and A go first (6 minutes). I comes back (3 minutes), then B and C go out (30 minutes). A then comes back (6 minutes), and finally I and A leave (6 minutes). The total time is 51 minutes.

10. Assume that the sphere is centred at the origin. Suppose point P_i has coordinates (x_i, y_i, z_i) for $i = 1, \ldots, n$. Then

$$\begin{aligned}(d_{ij})^2 &= (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \\ &= 2 - 2x_ix_j - 2y_iy_j - 2z_iz_j\end{aligned}$$

since $x_i^2 + y_i^2 + z_i^2 = 1$ for each i. But $d_{ij} = d_{ji}$ and $d_{ii} = 0$, so we get

$$\begin{aligned}\sum_{i<j} d_{ij}^2 &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^2 \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}(1 - x_ix_j - y_iy_j - z_iz_j) \\ &= n^2 - \Big(\sum_{i=1}^{n} x_i\Big)^2 - \Big(\sum_{i=1}^{n} y_i\Big)^2 - \Big(\sum_{i=1}^{n} z_i\Big)^2 \\ &\leq n^2.\end{aligned}$$

JUNIOR SECTION SOLUTIONS

1. Samos. 2. Rabbit. 3. 1990's. 4. Archimedes.

5. 8. (1c, 2c, 2c, 5c, 10c, 20c, 20c, 50c.)

6. Yes. First measure 12 kilos, then 6, then 3.

7. 553451234512345.

8. $a = 7$ and $b = 4$.

9. No. Represent each person by a dot on a page. Join two dots with a line if the corresponding people are related. If everyone is related to exactly three other people, that means the total number of lines must be $\frac{1}{2}(3 \times 2005)$, but this is not a whole number.

10. Apply the Arithmetic Mean-Geometric Mean inequality to $x^4, y^4, 9$ and 9, to get

$$\frac{x^4 + y^4 + 9 + 9}{4} \leq (x^4 + y^4 + 9 + 9)^{\frac{1}{4}}.$$

Equality is obtained when $x^4 = y^4 = 9$.

RÍOMHAIRE AGUS DÚCH

A Planning Sheet: R&D by Neil Hallinan

Which may be used with Junior Cert Foundation Mathematics

In the Junior Certificate 'Guidelines for Teachers' published by the NCCA there is a Planning and Record Sheet which outlines how items mentioned in the syllabus may be listed. The following tables follow that format and attempts to give as complete a picture as possible. This list is not to be taken as a direct copy of the Syllabus and is for guideline purposes only. I have added in some extra items which have appeared in text-books designed for the course as well as some items which must appear by implication even if they are not specifically mentioned in the syllabus. Please advise all Errors and Omissions to the Editor: hallinann@eircom.net .

Total List of Topics: Sets/ Numbers /Applied Arithemtic and measure: (%,Money,Measure)/ Statistics and data handling /Algebra/Relations, Functions, graphs/Geometry *items are not mentioned directly in the syllabus	1st Year	2nd Year	3rd Year
Sets and *symbols			
Elements ε			
List {a,b,c}			
Rule {x| x is … }			
Universal set U			
Subset ⊂			
Null set (empty set) Ø or { }			
Equality {1, 2, 3}= {1, 2, 3}			
Venn diagram			
Intersection (2 sets) ∩			
Union (2 sets) ∪			
Complement of a set A'			
Commutativity (for Intersection and Union)			
Algebra			
Formulae			
Idea of unknown			
Idea of variables			
Evaluation of linear expressions: Types: ax + by or a(x+y) (a, b, x, y in N)			
Evaluation of quadratic expression: $x^2 + ax + b$ (a, b, x in N)			
Simplify (using associativity and distribution) *Multiplication of Signs(no double negatives), Numbers and Letters a(x ± b) + c(x ± d) or x(x ± a) + b(x ± c) a,b,c,d,x in N			
Solve simple equations (first degree) : solution in N			

Numbers	1st Year	2nd Year	3rd Year
Use of Calculator			
*Digits			
Place Value			
Order - Less than, greater than, <, >, ≤, ≥			
*Numbers in words			
Natural numbers N – add, subtract, multiply, divide			
Priority of operations -*BOMDAS			
Multiples			
LCM			
Brackets – use of calculator, *change flat bracket to round			
Estimates – leading to approximate answers			
Powers – indices – a^n (n a Natural number, not zero)			
Integers Z			
Number Line			
Addition in Z: *Use of signed numbers			
Positive Rationals Q^+			
Fractions – Denominators: 2,3,4,5,7,8,10, 16,100, 1000 especially			
Equivalent fractions			
Fractions – add, subtract, multiply;			
Fractions - without a calculator: Denominators:2,4,5,8,10,100,1000			
Estimation			
Fractions – Decimal			
Decimal – Fraction			
Fraction – Percentage			
Percentage – Fraction			
Decimal – Percentage			
Percentage – Decimal			
Equivalence of Fraction/Decimal/Percentage			
Decimals: Place value			
Priority of operations			
Rounding (to not more than 3 decimal places)			
Estimation leading to approximate answers			
Squares			
Square roots			
Commutative property			

Relations, functions, graphs	1st Year	2nd Year	3rd Year
Arrow graph diagram to illustrate relations			
Couples			
*Venn diagram – arrow graph on a single set diagram			
*Identity couple			
*x-axis and y-axis			
Plot points			
Table of values : y = ax +b (values of a, b in N)			
Joining points to form a line			
Reading values from a graph: Simple interpretation			
Applied Arithmetic			
Percentages, Money			
Bills: Shopping			
Electricity/Gas: Meter reading			
Phone			
% of…			
VAT			
Fixed charges/Variable charges			
Increase by %..			
Decrease by %..			
% Profit/Loss: find % profit/loss from given CP and SP			
Find SP from given CP and %Profit/Loss			
%Discount			
Compound Interest (not more than three years)			
Income Tax/ *Credits			
*Wholesale/Retail			
*Service Charge			

Applied Arithmetic (continued): **Measurement:**	1st Year	2nd Year	3rd Year
SI units: Length: mm, cm, m, km			
Mass: g, kg (Weight)			
Area: cm^2, m^2, km^2			
Volume: cm^3, m^3 Litre (*l and ml)			
Time: Second (s), Minute, Hour			
Conversion of Units/ Use of Maths Tables			
24-hr clock/ Reading time on a clock/*12 hour clock			
*Length of time: How long?			
*Adding/Subtracting time: Use of DMS key			
Transport timetables			
Average Speed, Distance, Time relationship			
Distance from a map			
Scale on drawings			
*Problems of 'How much?', 'how many?' 'how many left?'			
Length: Perimeter			
Circumference $2\pi r$			
Radius/Diameter			
Length circumference ¸ length diameter = π			
Area: Square,			
Rectangle,			
Triangle			
Disc (circle) πr^2			
Volume: Rectangular Solid (Block)			
Cylinder $\pi r^2 h$			
*Compound figures			
*Tiling			
*Rotations/revolutions of a circle			

Statistics and data handling	1st Year	2nd Year	3rd Year
Collecting data			
Recording data			
Tabulating data			
Drawing pictograms			
Interpreting pictograms			
Drawing Bar-charts			
Interpreting Bar-charts			
Drawing Pie-charts Angles multiples of 30^o and 45^o			
Interpreting Pie-charts			
Drawing Trend Graphs			
Interpreting Trend graphs			
Sketching graphs and tables of data relationships			
Interpretation of sketches and tables			
Mean average of a list			
Mean average of a frequency table			
Mode			
Discrete array expressed as a frequency table			
Geometry			
The Plane			
*A point			
A line *ab*			
Line segment [*ab*]			
Length of a segment |*ab*|			
Angle : *a turn between half lines			
Naming an angle with 3 letters			
Measuring an angle : Protractor :			
Types of angles: Straight angle is 180º			
Acute angle			
Right angle			
Obtuse angle			
Vertically opposite angles are equal			
Parallel lines			
Perpendicular lines			

Geometry (continued):	1st Year	2nd Year	3rd Year
Use of set square/compass/straight-edge/ruler			
How to measure and draw with a ruler			
How to draw with a set-square			
How to measure and draw with a compass			
Measuring the perimeter of a square			
Measuring the perimeter of a rectangle			
Types of triangles: Scalene			
Isosceles			
Equilateral			
Types of quadrilaterals (convex): any 4 sides			
Parallelogram			
Rectangle			
Square			
Constructions:			
Construct a line segment of given length			
Construct a triangle: SSS – given			
Construct a triangle : SAS - given			
Construct a triangle: ASA - given			
Construct a triangle: 90º angle + sufficient other data			
Construct a rectangle, a square			
Draw a line through a point parallel to a given line			
Divide a line segment into 2 equal parts			
Divide a line segment into 3 equal parts			
Bisect an angle without a protractor			
Distance of a point from a line (meaning)			
'Base' and 'perpendicular height' for a parallelogram			
Three angles in a triangle sum to 180º			
Pythagoras Theorem for a right-angled triangle			
*Use of calculator for square and square root			
A diagonal bisects the area of a rectangle			
Area of a parallelogram = base length x perpendicular height			
Transformation Geometry:			
Central symmetry			
Axial symmetry			
Construct an image under Axial symmetry: rectilinear figures only			
Construct an image under Central symmetry: rectilinear			

MATHS IN PRACTICE

I stared at the light as it hovered above me – a strange oval shape with darker markings appearing at regular intervals. My full attention was directed towards it. Was it an alien craft of some sort? Suddenly it moved. Quickly, it came nearer my face. I gulped. My mouth was open – what was going to happen? 'A little wider, please!' The sonorous tones of a voice sounded in my ear. I knew that voice – it was my dentist! The light was no alien craft but it helped to imagine it as such – anything to provide a distraction.

'I think I'll do an X-ray', he said, 'just to make sure. And what are your plans for the holidays?'
'Ah onh owe eh', I valiantly ventured.
'That's great', he said, 'Close now, and swallow.'
'You must enjoy teaching mathematics!' he continued. I could speak again. 'Of course! I teach the future dentists. Tell me, where does a dentist use maths?'
'Open', came the command again. 'Ratios', he said after a little thought.
'Ow owe?', I uttered.
'Well, in this X-ray, for example', he replied as though my speech had been with full clarity. 'An X-ray may be taken at an angle which gives a picture which has measurements different from the real measurements of the tooth, or it may be shown on an enlargement and so on. When dentists do a root canal treatment, for instance, they have to place a marker probe of definite length into the tooth root and then take the X-ray.'

I quickly thought of the formula which they must use:

Picture length of marker: Actual Length of marker = Picture length of tooth root: Actual length of tooth root

Dentists would need to be sure of their maths and their measurements, especially with the word 'drill' involved. And it was something we teach quite a lot at school – ratios, that is.

The X-ray's were completed and my treatment continued. Dr. Kevin worked at my teeth as a plasterer would work on dry-lining or as an artist on a canvas – palette-knife flickering with practiced dexterity.

'There's more.' he said. My heart sank. More visits, more drills.
'Oh ads ah?', I feebly croaked.
'We have to work with precision angle measurements', he said, oblivious to my sweat. I relaxed again.
'Right up to 6º off the tooth – at least according to one authority in the past, but that requirement was relaxed later', he explained without haste. I thought, 6º is not something we measure everyday at school, right enough.

'And, of course,' he said as he stepped back from behind my head, 'the finished product should adhere to the proportions of the Golden Ratio – just as you will when I am finished!'
He sounded delighted and I was reassured that I was in very capable hands – both of an artist and a dentist well-grounded in his mathematics.

'Thank you very much, Dr. Kevin,' I said, as I stood up to leave. 'How long until my next visit?'
'Well, perhaps you could figure that out for me,' he said with a twinkle. 'A little bit of brushing, a little bit of flossing, and of course, statistics, as you should know as a mathematician.'

I came away feeling very much enlightened. That conversation helped to give a meaning to some of the humdrum work that I do every day as a teacher. From simple ratios to complicated statistics or probability the classroom could move beyond its boundaries and I would continue to teach the future dentists with enhanced assurance and a perfectly proportioned smile.

Neil Hallinan
With thanks to Dr Kevin.