



BABHTA 1

ROUND 1

Q1.1) In a given triangle ABC, using the usual notation,  
side  $a = 3$  cm, side  $b = 4$  cm and side  $c = 5$  cm  
Find the numerical value of  $\sin(A) + \sin(2B) + \sin(3C)$ .

Answer in form  $\frac{a}{b}$ , where  $a$  and  $b \in \mathbb{N}$ , in simplest form.

Q1.2) Find, in terms of  $n$ , the value of  ${}^n C_1 + 6({}^n C_2) + 6({}^n C_3)$

Answer in simplest form



BABHTA 2

ROUND 2

Q2.1) Four integers are added to the set  $\{3, 4, 5, 5, 8\}$  increasing the mean, median and mode each by 1.

Write down the elements of the new set **in ascending order**.

Q2.2) What is the value of the base **b**, in surd form, when

$$\log_b(10) + \log_b(10)^2 + \log_b(10)^3 + \dots + \log_b(10)^{10} = 110$$



BABHTA 3

ROUND 3

Q3.1) Find the value of the integral  $\int (1 + \sin(3x)) dx$

Q3.2) Find the 5<sup>th</sup> term of the expansion  $(x-3y)^{12}$ .  
Answer in simplest form  $ax^py^q$ , where  $a, p, q \in \mathbb{N}$ .



BABHTA 4

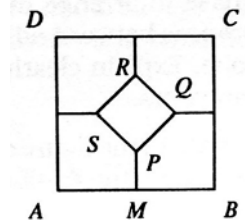
ROUND 4

Q4.1) ABCD and PQRS are concentric squares. The diagonals of PQRS are parallel to the sides of ABCD.

If  $|AB|$  is 1 cm and  $|PQ| = \frac{1}{2}$  cm, calculate

the length of  $[PM]$ ,

where M is the midpoint of the line segment  $[AB]$ .



Answer in form  $\frac{a - \sqrt{b}}{c}$ , where  $a, b$  and  $c \in \mathbb{N}$ ,  
in simplest form.

Q4.2) Find all the cube roots of  $4 + 4\sqrt{3}i$ .

Answers in the form  $r(\cos(A) + i\sin(A))$ , where A is in degrees.



BABHTA 5

ROUND 5

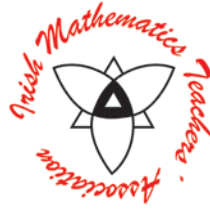
Q5.1) Given that  $a$  and  $b \in \mathbb{R}$ ,  $0 < a < b$  and  $a^2 + b^2 = 6ab$   
find the numerical value of  $\frac{a+b}{a-b}$ .

Answer in simplest surd form.

Q5.2) Solve for  $x$  and  $y$

$$\begin{aligned}x(x+y) &= 108 \\y(x+y) &= \frac{297}{4}\end{aligned}$$

Answers in the form  $(x, y)$  where  $x$  and  $y \in \mathbb{Q}$



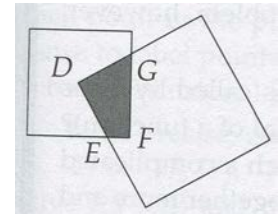
BABHTA 6

ROUND 6

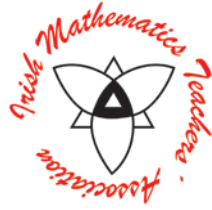
- Q6.1) A jar has 499 fair coins and one similar coin with heads on both sides.  
A coin is chosen from the jar at random and flipped 9 times.  
Given that it comes up heads every time, what is the probability that the coin is the two headed coin?

Answer in form  $\frac{a}{b}$ , where  $a$  and  $b \in \mathbb{N}$ .

- Q6.2) A square of side 3 m and a square of side 4 m overlap as shown in the diagram.  
D is the centre of the 3 m square.  
Find the area of the shaded region DGFE.



Answer in the simplest form  $\frac{a}{b}$ , where  $a$  and  $b \in \mathbb{N}$ .



BABHTA 7

ROUND 7

Q7.1) Solve for  $x$  :  $|x^2 - 4x + 5| \leq 6$ , where  $x \in \mathbb{R}$ .

Answer in the form  $a \leq x \leq b$  in simplest surd form.

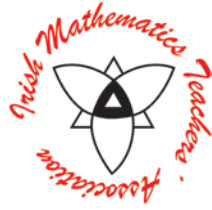
Q7.2) If  $\frac{5(10^{27}-1)}{9}$  is written as an integer how many times does 5 appear in the answer?

Q7.3) Two vertices of an equilateral triangle lie on the line  $y = 2x - 2$ , and the third vertex lies on the line  $y = 2x + 2$ .  
Find the area of the triangle.

Answer in form  $\frac{a\sqrt{b}}{c}$ , where  $a, b$  and  $c \in \mathbb{N}$ .

Q7.4) Three fair 6-sided dice are thrown.  
What is the probability that the three numbers rolled are three consecutive numbers, in some order?

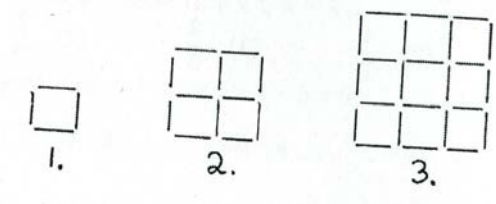
Answer in the simplest form  $\frac{a}{b}$ , where  $a$  and  $b \in \mathbb{N}$ .



BABHTA 8

ROUND 8

Q8.1) Square grid patterns of matches are shown, namely a 1-square, a 2-square and a 3-square.



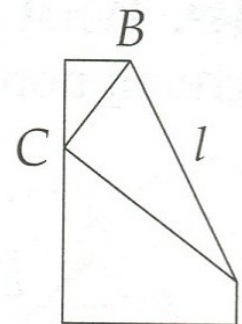
How many matches must be added to a 500-square to make a 501-square?

Q8.2) Find all the complex numbers such that  $|z - 1| = |z + 3| = |z - i|$ , where  $z = x + iy$ .

Answers in the form  $a + ib$  where  $a$  and  $b \in \mathbb{R}$ .

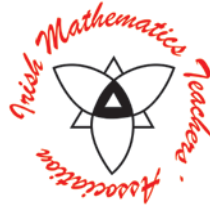
Q8.3) Find the values of  $x$  and  $y$  for which  $x^2 + 6x + y^2 = 4$ .  
 Answers in the form  $(x, y)$ , where  $x$  and  $y \in \mathbb{Z}$ .

Q8.4) A corner of a rectangular piece of paper of width 8 cm is folded over so that it coincides with the point  $C$  on the opposite side. Given that  $|BC| = 5$  cm, calculate, in cms, the length of the fold  $l$ .



Answer in form  $a\sqrt{b}$ , where  $a$  and  $b \in \mathbb{N}$ .

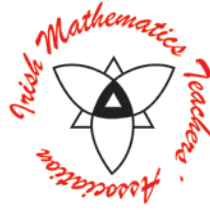




## SCOILT

## TIEBREAK

- T1) Find all the numerical values of  $x$  for which  $7^{2x} = 45(7^x) + 4(7^2)$
- T2) What is the relationship between  $a$  and  $b \in \mathbb{R}$  for which the line  $x + y = a$  is a tangent to the circle  $x^2 + y^2 = b$ ?
- T3) What is the remainder when  $8^{2020}$  is divided by 9?
- T4) John will need €7000 in three years time to purchase a car.  
How much should he deposit at the end of each month into an account that pays 6% EAR to achieve his goal?  
Answer to the nearest euro.
- T5) Find the greatest integer,  $x$ , for which  $3^{20} > 32^x$ ?
- T6) Find the acute angle between the lines  $2x - y = 3$  and  $x - 3y = 4$ .
- T7) Find the area enclosed in the graph  $x^2 + y^2 = 16x + 32y$ .  
Answer in terms of  $\pi$ .
- T8) Given  $1.2 \leq a \leq 5.1$  and  $3 \leq b \leq 6$  find the highest possible value for the quotient  $\frac{a}{b}$ .
- T9) Find all the ordered pairs of real numbers  $(x, y)$ , such that  
 $x^3 - y^3 = 19$  and  
 $x^2y - xy^2 = 6$
- T10) Find the area of the triangle with vertices  $(5,4)$ ,  $(3,6)$  and  $(2,1)$



## Answer Key Regional Round

### Round 1

Q1.1  $\frac{14}{25}$

Q1.2  $n^3$

### Round 2

Q2.1 {3,4,5,5,6,6,6,8, 11}

Q2.2  $b = \sqrt{10}$

### Round 3

Q3.1  $x - \frac{\cos(3x)}{3} + C$

Q3.2  $40095 x^8 y^4$

### Round 4

Q4.1  $\frac{2-\sqrt{2}}{4}$

Q4.2  $2(\cos(20^\circ) + i\sin(20^\circ)), 2(\cos(140^\circ) + i\sin(140^\circ)), 2(\cos(260^\circ) + i\sin(260^\circ))$

### Round 5

Q5.1  $-\sqrt{2}$  (only)

Q5.2  $(8, \frac{11}{2}), (-8, -\frac{11}{2})$

### Round 6

Q6.1  $\frac{512}{1011}$

Q6.2  $\frac{9}{4}$

### Round 7

Q7.1  $2 - \sqrt{5} \leq x \leq 2 + \sqrt{5}$

Q7.2 27

Q7.3  $\frac{16\sqrt{3}}{15}$

Q7.4  $\frac{1}{9}$



Round 8

Q8.1 2004

Q8.2  $-1 - i$

Q8.3  $(0, \pm 2), (-6, \pm 2), (-1, \pm 3), (-5, \pm 3)$

Q8.4  $5\sqrt{5}$

Tiebreak

T1 2

T2  $b = \frac{a^2}{2}$  or  $a = \pm\sqrt{2b}$

T3 1

T4 €178

T5 6

T6  $45^\circ$

T7  $320\pi$

T8 1.7

T9  $(3, 2)$  and  $(-2, -3)$

T10 6