



Craobhchomórtas Foireann Mata 2018
Team Maths Final 2018



Babhata 1

Round 1

Q1.1) A triangle has angles of 30° and 45° . If the side opposite the 45° angle has length 8 cm. Calculate the length of the side opposite the 30° angle.

Answer in form $a\sqrt{b}$, where a and $b \in \mathbb{N}$

Q1.2) Find the equations of the tangents to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ which are parallel to the line $4x + 3y + 5 = 0$.

Answers in form $ax + by + c = 0$, where a, b and $c \in \mathbb{Z}$



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Babhta 2

Round 2

- Q2.1) For $x \in \mathbb{R}$, find the range of values for which
 $1 \leq |x - 2| \leq 7$
- Q2.2) An equilateral triangle is drawn with side length **a**. A new equilateral triangle is formed by joining the midpoints of the sides of the first one. Then a third equilateral triangle is formed by joining the midpoints of the sides of the second one; and so on forever. In terms of **a** what is the limit of the sum of the perimeters of all the triangles drawn?



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Babhta 3

Round 3

Q3.1) Solve for x and $y \mid \mathbb{N} : 2^{2x} - 3^{2y} = 55$.

Answer in form (x, y) .

Q3.2) If $\frac{x}{a-b} = \frac{y}{b-c} = \frac{z}{c-a}$, find the numerical value of $x + y + z$, where a , b and c are distinct real numbers.



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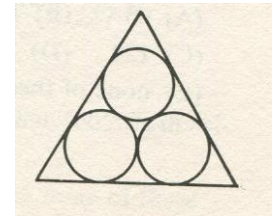


Babhta 4

Round 4

- Q4.1) An open box with volume 24 cm^3 is to be made from a square piece of tin by cutting 2 cm from each corner and turning up the sides.
Calculate the length of the side of the square piece required.
Answer correct to 3 decimal places.

- Q4.2) Each of the three circles in the adjoining figure is externally tangent to the other two, and each side of the triangle is tangent to two of the circles.
If each circle has a radius 3 cm , calculate the perimeter of the triangle.



Answer in simplest surd form.



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Babhla 5

Round 5

Q5.1) Given $g(x) = 1 - x^2$ and $f(g(x)) = \frac{1 - x^2}{x^2}$ when $x \neq 0$,
find the numerical value of $f\left(\frac{1}{2}\right)$

Q5.2) A set of consecutive positive integers beginning with 1
is written on a blackboard.
One number is erased.
The average of the remaining numbers is $35\frac{7}{17}$.
How many numbers were in the set and what number
was erased?



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Babhtha 6

Round 6

Q6.1) Find **ALL** the values of x , y and z which satisfy the following equations:

$$y + z = yz - 1$$

$$z + x = zx - 2$$

$$x + y = xy - 5$$

Q6.2) The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle.

Find the cosine of the smallest angle.

Answer in the form $\frac{a}{b}$, where $a, b \in \mathbb{N}$.



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Babhta 7

Round 7

Q7.1) Given that $\frac{3x^2 - 10x - 2}{(x-1)(x-2)(2x+1)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{2x+1}$

find the numerical value $A + B + C$.

Q7.2) Solve for x : $\sqrt{x^2 - 7x + 10} - \sqrt{x^2 + x - 6} = x - 2$

Q7.3) Given $0^\circ \leq x \leq 90^\circ$ find all the values of x for which
 $\sin(7x)\cos(2x) = \sin(2x)\cos(3x)$

Q7.4) Find the values of m for which the quadratic equation

$$12(m+2)x^2 - 12(2m-1)x - 38m - 11 = 0$$

will have equal roots.



Babhta 8

Round 8

Q8.1) Two identical jars are filled with alcohol solutions, the volume of alcohol to the volume of water being $p : 1$ in one jar and $q : 1$ in the other jar. If the entire contents of the two jars are mixed together, calculate the ratio of the volume of alcohol to the volume of water in the mixture.

Answer as a fraction in terms of p and q .

Q8.2) Given that $i^2 = -1$, for how many integers n is $(n+i)^4$ an integer?

Q8.3) Calculate, in terms of n , the sum of the first n terms of the series

$$1 + (1 + 2) + (1 + 2 + 4) + (1 + 2 + 4 + 8) + \dots + (1 + 2 + 4 + \dots + 2^{n-1})$$

Answer in terms of n in simplest form, without brackets.

Q8.4) A box contains two 1 cent coins, four 5 cent coins and six 10 cent coins.

Six coins are drawn without replacement, each coin having an equal probability of being chosen.

What is the probability that the value of the coins drawn is at least 50 cents?

Answer in simplest form $\frac{a}{b}$, where a and $b \mid N$.



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Tiebreak

T1 Find ALL solutions to the equations $\cos^2(x) = \frac{3}{4}$, in the range $-90^\circ < x < 180^\circ$.
Answers in degrees.

T2 Find the range of values of $x \in \mathbf{R}$ for which

$$\frac{3x}{2x - 3} > 6.$$

T3 If p leaves a remainder of 3 when divided by 5 and q leaves a remainder of 2 when divided by 5, what is the remainder when the product pq is divided by 5?
 p and q are positive integers.

T4 Given that a circle and a square are of equal area calculate the ratio of the circumference of the circle to the perimeter of the square.
Answer, as a percentage, correct to one decimal place .

T5 Toy cars A, B, C, D and E are arranged in order of increasing height with A being the smallest and E being the largest. The heights are all consecutive even integers and, when added their sum is 450.

Find the value of $\frac{2E - 3C + A}{4B - 3D}$.

Answer in simplest form $\frac{a}{b}$, where a and $b \in \mathbf{N}$.

T6 Find all the values of $x \in \mathbf{R}$ where $(2x - 5)^3 + 6(2x - 5) = 5(2x - 5)^2$.

Answers in simplest form $\frac{a}{b}$, where a and $b \in \mathbf{N}$.

T7 Two angles in a triangle measure 25° and 85° . The longest side measures 10 cm.
Find the length of the shortest side correct to two decimal places.

T8 Express $\frac{5}{2 - \sqrt{3}}$ in the form $a(b + \sqrt{c})$ where a, b and $c \in \mathbf{N}$.



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T9 The points $(1, -1)$, $(a, 2)$ and $(b, 1)$ are collinear.
Express a in terms of b .

T10 Solve for x : $27^{2x-1} = 81^{x+2}$.

Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

T11 A three-digit integer contains one of each of the digits 2, 6 and 5.
What is the probability that the integer is divisible by 5?

Answer in simplest form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

T12 How many integer values of k are there such that the graphs of $y = x^2$ and $y = kx - 1$ do not intersect?

T13 If the graph of the equation $y = (x + 2)^2$ is reflected in the y -axis, what is the equation of the resulting graph?

Answer in form $y = ax^2 + bx + c$, where a, b , and $c \in \mathbb{Z}$.

T14 Find the perpendicular distance from the point $(4, -3)$ to the line $y = -2x - 4$.

Answer in form $\frac{a\sqrt{b}}{b}$, where a and $b \in \mathbb{N}$.

ANSWER KEY : TEAM MATHS FINAL 2018

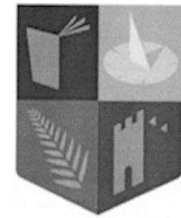
Round 1

Q1.1 $4\sqrt{2}$ Q1.2 $4x + 3y + 19 = 0$ and $4x + 3y - 31 = 0$

Round 2



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Q2.1 $-5 \leq x \leq 1$ and $3 \leq x \leq 9$ Q2.2 6a

Round 3

Q3.1 (3, 1) Q3.2 0

Round 4

Q4.1 7.464 Q4.2 $18\sqrt{3} + 18$

Round 5

Q5.1 1 Q5.2 69 Numbers, 7 erased.

Round 6

Q6.1 $x = 4$ or -2 , $y = 3$ or -1 , $z = 2$ or 0 Q6.2 $\frac{3}{4}$

Round 7

Q7.1 2 Q.2 2
Q7.3 $0^0, 36^0, 72^0, 22.5^0, 67.5^0$. Q7.4 $-\frac{1}{2}$ OR -1

Round 8

Q8.1 $\frac{p+q+2pq}{p+q+2}$ Q8.2 3 Q8.3 $2^{n+1} - n - 2$ Q8.4 $\frac{127}{924}$