



BABHTA 1

ROUND 1

Q1.1) $z = a + ib$, where a and $b \in \mathbb{R}$ and $i = \sqrt{-1}$.
If $f(z) = z^2$, calculate $f(3 + 4i)$

Answer in form $a + ib$, where a and $b \in \mathbb{Z}$

Q1.2) ABCD is a square of side a and [BC] is produced to E
so that $|CE| = \frac{1}{2}|BC|$.

Calculate $|AE|^2 - |DE|^2$ in terms of a .

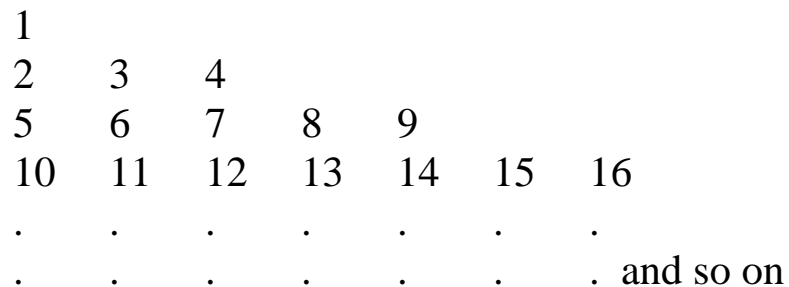


BABHTA 2

ROUND 2

Q2.1) In how many ways can three of the letters A, B , C, a , b , c, d be arranged in a row, using one capital and two small letters, so that the capital letter is always first, if each letter is used only once?

Q2.2) The positive integers are arranged in a triangular grid as shown.



What is the fourth number in the 60th row?



BABHTA 3

ROUND 3

Q3.1) Find the equation of the line through (1,4) which is parallel to the line $2x - y - 3 = 0$.

Answer in the form $ax + by + c = 0$,
where a , b and $c \in \mathbb{Z}$.

Q3.2) If $x^2 + y^2 = 10$ and $xy = 3$ find all possible values of $x + y$.



BABHTA 4

ROUND 4

Q4.1) $x^4 - 48x + 28 = (x^2 + ax + 2)(x^2 + 4x + b)$.

Find the values of a and b, where a and b \in R.

Q4.2) A triangle ABC has vertices A(18 , - 4), B(13 , 6), and C(x, 12).

The area of the Δ ABC is 50 square units.

Find all possible points C in form (x , y).

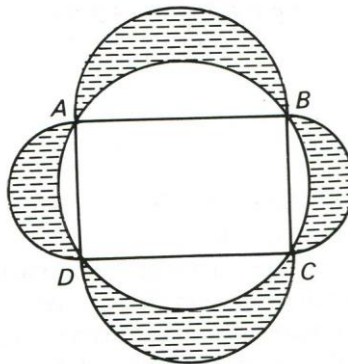


BABHTA 5

ROUND 5

- Q5.1) A club identifies each of its members with a unique three-digit number, the sum of whose digits is 24. How many such numbers exist?

- Q5.2)



ABCD is a rectangle such that $|AB| = 8$ cm and $|BC| = 6$ cm. Semicircles are drawn on the four sides of the rectangle ABCD, each semicircle lying outside the rectangle. The circle passing through A, B, C and D determines four crescent shaped regions, one inside each semicircle as shown in the diagram. Find the total area of the four crescent shaped regions in simplest form.



BABHTA 6

ROUND 6

Q6.1) Solve for a and b :

$$\log(8x^3 + 4x^2 - 2x - 1) = \log(2x-1) + 2\log(ax+b).$$

Q6.2) A dice rests on a table. Sam can see three faces and a total of 9 spots. Pat, who is on the opposite side of the table, can see three faces and 15 spots.
How many spots are on the top face of the dice?



BABHTA 7

ROUND 7

Q7.1) f and g are functions such that $f(x) = ax + 5$ and $g(x) = cx - 3$, where a and $c \in \mathbb{N}$.
If $f(g(x)) = g(f(x))$ calculate the values of a and c .

Q7.2) Given two different two-digit positive integers whose product is 2016.

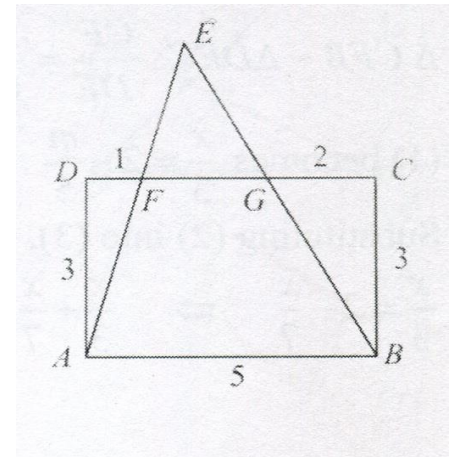
Find the greatest possible sum of the four digits.

Q7.3) In the expansion $(x + 3y + \frac{z}{4})^3$ find the coefficient of yz^2 .

Answer in form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

Q7.4) In the rectangle $ABCD$. $|AB| = 5$
and $|BC| = 3$.
Points F and G are on $[CD]$ so that
 $|DF| = 1$ and $|GC| = 2$.

AF and BG intersect at E .
Calculate the area of the triangle AEB .



Answer in form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.



BABHTA 8

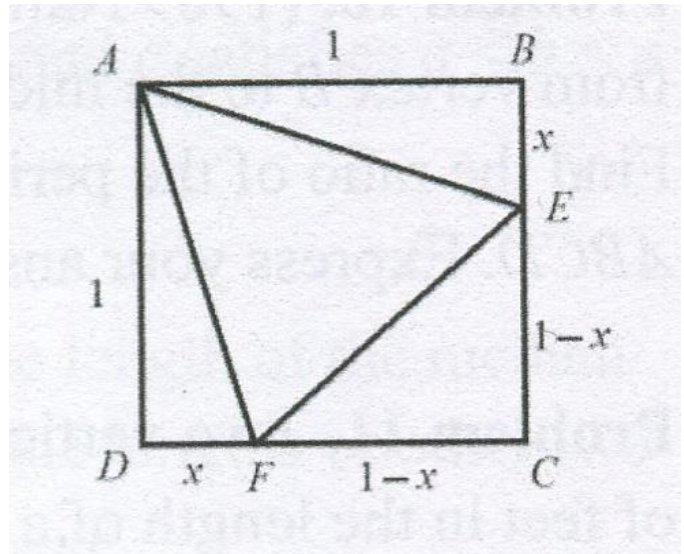
ROUND 8

Q8.1) If $0^\circ \leq x \leq 180^\circ$ and $0^\circ \leq y \leq 180^\circ$ find the pairs of angles, in the form (x,y) , which satisfy the equations :

$$\sin(x + y) = +\frac{1}{\sqrt{2}}$$

$$\cos(2x) = -\frac{1}{2}$$

Q8.2) In the diagram an equilateral triangle is inscribed in a square of side 1 unit. From the diagram use the value of x to find the area of the triangle in form $a\sqrt{b} - c$, where a, b and $c \mid \mathbb{N}$.



Q8.3) The coordinates of the vertices of a cyclic quadrilateral are $A(2, 5)$, $B(-4, 0)$ and $D(7, 0)$. If the angles at B and D are right angles calculate the coordinates of vertex C .

Q8.4) Solve for x : $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$.

TIEBREAK

- T1) The price of coat was reduced by 40% and the new price was then reduced by a further 40% during a sale.
By what percentage was the original price reduced?
- T2) The sides of a triangle ABC are 10, 24 and 26 cm long.
A rectangle, with width 3cm, has an area equal to that of the triangle.
Find the perimeter of the rectangle, in cms.
- T3) If $0^\circ \leq x \leq 360^\circ$ find the values of x for which
 $3\sin^2(x) - \sin(x)\cos(x) - 4\cos^2(x) = 0$.
Answers to nearest degree.
- T4) Find the common ratio of the geometric sequence where the first term exceeds the third by 60 and the sum of the first two terms is 48.

Answer in form $\frac{a}{b}$, where a and b $\in \mathbb{Z}$.

- T5) Find the points of intersection of the circles:

$$x^2 + y^2 + 4x - 2y - 5 = 0 \text{ and } x^2 + y^2 + 2x - 7 = 0$$

Answers in coordinate form (x , y).

- T6) A train travelling at 60 km/h takes 15 seconds to pass a certain point.
How long is the train?
- T7) A complex number in the form $a + ib$, when multiplied by its conjugate $a - ib$ yields 13, where a and b $\in \mathbb{R}$.
Find the numerical value of $a^4 + 2a^2b^2 + b^4$.

- T8) Write in simplest form:

$$((x + 1)^2(x - 1)^2)((x^2 + 1)^2)$$

- T9) The members of a Team Maths 2017 team are chosen at random from a class of twelve boys and fifteen girls.
Find the probability that the team consists of two boys and two girls.

Answer in simplest form $\frac{a}{b}$, where a and b $\in \mathbb{N}$.

T10) Simplify
$$\frac{\frac{x}{1+x} + \frac{1-x}{x}}{\frac{x}{1+x} - \frac{1-x}{x}}$$

Answer in simplest form as a fraction in terms of x.

T11) Find the perimeter of a right-angled triangle whose hypotenuse is 2 units and whose area is 1 square unit.

Answer in form $a + b\sqrt{c}$, where a, b and c $\in \mathbb{N}$.

T12) $(m, 3)$ and $(1, m)$ are two points on the line l. The slope of l is m. Given that $m > 0$, find the value of m in surd form.

T13) The point with coordinates $(p\cos(A), p\sin(A))$ lies on the circle $x^2 + y^2 = 16$.

Find the value of p, where $p > 0$.

Answer key , Round 1 2017

Round 1	Q1.1) $-7 + 24i$	Q1.2) $2a^2$
Round 2	Q2.1) 36	Q2.2) 3485
Round 3	Q3.1) $2x - y + 2 = 0$	Q3.2) ± 4
Round 4	Q4.1) $a = -4$, $b = 14$	Q4.2) (20, 12) and (0,12)
Round 5	Q5.1) 10	Q5.2) 48 cm^2
Round 6	Q6.1) $a=2$, $b=1$	Q6.2) 5
Round 7	Q7.1) $a = 1$, $c = 1$	Q7.2) 20
	Q7.3) $\frac{9}{16}$	Q7.4) $\frac{25}{2}$
Round 8	Q8.1) $(60^0, 75^0)$ and $(120^0, 15^0)$	Q8.2) $2\sqrt{3} - 3$
	Q8.3) (1, -6)	Q8.4) 2

Tiebreak

T1) 64%	T2) 86	T3) $53^0, 233^0, 135^0, 315^0$
T4) $-\frac{1}{4}$	T5) (-3,-2) and (1,2)	T6) 250m
T7) 169	T8) $(x^4 - 1)^2$ or $x^8 - 2x^4 + 1$	
T9) $\frac{77}{195}$	T10) $\frac{1}{2x^2 - 1}$	
T11) $2 + 2\sqrt{2}$	T12) $\sqrt{3}$	T13) 4