ROUND ONE

Q. 1 Solve the simultaneous equations:

$$3x + y = 5$$
$$x^2 + y^2 = 25$$

Q. 2 Find the equation of the circle which has $[AB]$ as a diameter, where A and B are the points $(-2, 1)$ and $(6, -3)$ respectively.

Write your answer in the form $x^2 + y^2 + 2gx + 2fy + c = 0, \quad g, f, c \in Z$.

ROUND TWO

Q. 1 Given that α and β are the roots of the equation $2x^2 - 6x + 1 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ in its simplest form.

Q. 2 Find the measure of the acute angle between the line l: $2x + y - 3 = 0$ and the line k: $x - 3y + 2 = 0$.

Give your answer correct to the nearest degree.

ROUND THREE

Q. 1 Write $-\sqrt{3}-i$ in general polar form.

Q. 2 The perpendicular distance from the point $(k, 3)$ to the line $3x + 4y - 2 = 0$ is 1 unit, where $k \in Q$.

Find the values of k.

ROUND FOUR

Q. 1 Find the values of A for which

$$\sqrt{3}\tan A = -1, \text{ where } 0^{\circ} \le A \le 360^{\circ}.$$

Q. 2 Find the range of values of x that satisfies the inequality

$$\frac{x-2}{x+3} \ge \frac{1}{2}, \quad x \ne -3.$$

Team Number ➔

NB. For this Round 5, answer the questions on **this page** and hand it up to the markers.

ROUND FIVE

Q. 1 Write the equation of the tangent to the curve

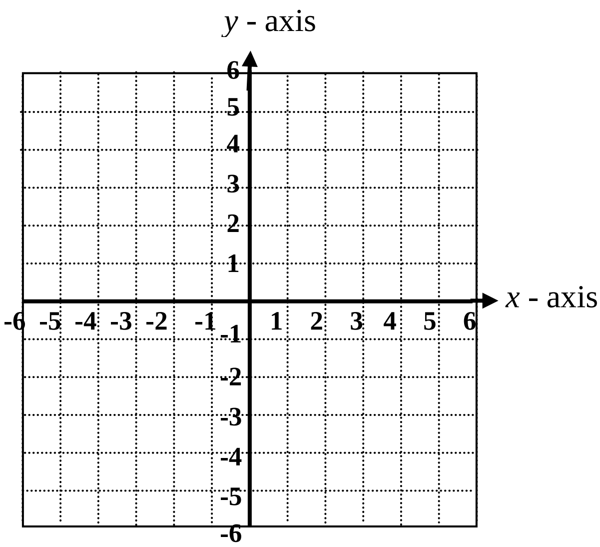
$$x^2 + y^2 + x = 1 \text{ at the point } (0, 1)$$

in the form $ax + by + c = 0$, where $a, b, c \in Z$.

Answer to Q. 1 ______________________________

Q. 2 Within the limits of the given diagram, clearly indicate the set of points that satisfy the inequality

$$2x + y \leq 4.$$



ROUND SIX

Q. 1 In the National Lottery, forty five balls numbered 1 to 45 are placed in a drum. Six balls are taken at random from the drum as the winning combination. If you select six numbers from 1 to 45, what is the probability that, at most, only one of them matches with the winning combination? Give your answer in decimal form, correct to two decimal places.

Q. 2 Find the value of x and the value of y for which

$$\begin{pmatrix} 3 & -5 \\ 5 & -8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 18 \end{pmatrix}.$$

ROUND SEVEN

Q. 1 Find all the values of x that satisfy the equation

$$\sin 4x + \cos 2x = 0, \text{ where } 0^\circ \le x \le 180^\circ.$$

Q. 2 A, B and C are points on the same level ground.
$|AB| = 12$ m and $|\angle CAB| = 90^\circ$.
A vertical pole is erected at C.
The angles of elevation of the top of the pole from A and B are 60° and 30° respectively.

Calculate $|AC|$ in metres, correct to one decimal place.

Q. 3 Find the equations of the asymptotes of the graph of

$$f(x) = \frac{x-3}{2x+5}.$$

Q. 4 Evaluate $\displaystyle\int_1^9 \frac{x-1}{x+\sqrt{x}}\,dx$.

ROUND EIGHT

Q. 1 Solve the equation

$$2\log_3 x - \log_3(x-2) = 2.$$

Q. 2 Mary is playing cards with Liam and Sam. She has nine clubs in her hand. The other four clubs, the Ace, 2, 3 and 4 are randomly dealt to Liam and Sam. Liam holds at least two of the four cards and may have all four.

If Liam shows Mary that he has the 2 of clubs, what is the probability that Sam has the Ace of clubs?

Q. 3 Simplify

$$\underset{h \to 0}{Lim}\left(\frac{\tan(x+h) - \tan x}{h}\right).$$

Q. 4 P is the set of lines $3x + y - t = 0$ for all real values of t.
Q is the set of lines $kx - 2y - 8 = 0$ for all real values of k.

Write the equation of the line that is common to the sets P and Q in the form

$$ax + by + c = 0, \text{ where } a, b, c \in Z.$$

Team Number _________

TIE BREAK 1

Write the answers on this page and hand it up

Q. 1 Evaluate $\int_0^{\pi} \frac{\sin^2 x}{1+\cos x}\,dx$.

Answer Q 1 __________________

Q. 2 Solve the equation

$$3^{2x} + 8(3^x) - 9 = 0.$$

Answer Q 2 __________________

Q. 3 For what range of values is $\tan^{-1}(x)$ defined for all real values of x?

Answer Q 3 __________________

Q. 4 Using only digits from the set {2, 3, 4, 5}, how many even numbers can be made, if no two digits in any number are the same?

Answer Q 4 __________________

Team Number __________

TIE BREAK 2

Write the answer on this page and hand it up

Find $$\frac{d\cos^2 x}{dx} + \frac{d\sin^2 x}{dx}$$

Answer ___________________

Answers

Round 1: Q. 1 $(0, 5), (3, -4)$

Q. 2 $x^2 + y^2 - 4x + 2y - 15 = 0$

Round 2: Q. 1 6

Q. 2 82°

Round 3: Q. 1 $2[\cos(^{7\pi}/_6 + 2n\pi) + i\sin(^{7\pi}/_6 + 2n\pi)]$ or

$2[\cos(210^\circ + n360^\circ) + i\sin(210^\circ + n360^\circ)]$

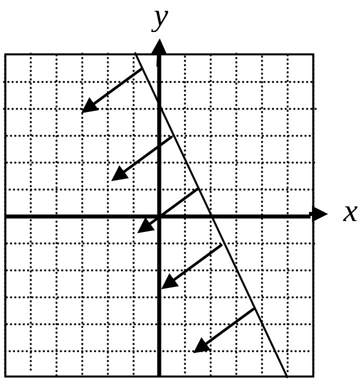
Q. 2 $-5, \; -{}^5/_3$

Round 4: Q. 1 $150^\circ, 330^\circ$ OR $^{5\pi}/_6, {}^{11\pi}/_6$

Q. 2 $\{x \mid x < -3\} \cup \{x \mid x \geq 7\}$

Round 5: Q. 1 $x + 2y - 2 = 0$

Q. 2



Round 6: Q. 1 0·82

Q. 2 $(2, -1)$

Round 7: Q. 1 $45^\circ, 105^\circ, 135^\circ, 165^\circ$

Q. 2 4·2 m

Q. 3 $x = -{}^5/_2, \; y = ½$

Q. 4 4

Round 8: Q. 1 3, 6

Q. 2 $^3/_7$

Q. 3 $\sec^2 x$ or $\dfrac{1}{\cos^2 x}$

Q. 4 $3x + y + 4 = 0$

Tie Break 1: Q. 1 π

Q. 2 0

Q. 3 $-90^\circ < \tan^{-1}(x) < 90^\circ$ or $-{}^\pi/_2 < \tan^{-1}(x) < {}^\pi/_2$

Q. 4 32

Tie Break 2: 0