ROUND ONE

Q. 1 Express $\frac{m}{m-n}+\frac{n}{n-m}$ in its simplest form.

Q. 2 The roots of $3x^2+5x+4=0$ are α and β.

Find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$.

ROUND TWO

Q. 1 Write $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{20}$

in the form $\frac{x}{2} + i\frac{y}{2}$, where $x, y \in \boldsymbol{R}$, using surd values where relevant.

Q. 2 Find the values of x which satisfy the equation

$\sin x = \sqrt{3}\cos x$, where $0° \le x \le 360°$.

ROUND THREE

Q. 1 Find the range of values of $\boldsymbol{x}$ that satisfy the inequality

$$\frac{x+5}{x-2} > \frac{1}{2}, \qquad x \neq 2.$$

Q. 2 Find the equations of the two tangents to the circle

$$x^2 + y^2 - 6y - 8 = 0$$

which are parallel to the line $4x - y = 0$.

Write your answers in the form $ax + by + c = 0$, where $a, b, c \in \boldsymbol{Z}$

ROUND FOUR

Q. 1 $z = -1 + 3i$ satisfies the equation

$$z^2 - z - 4iz + 5i - 5 = 0$$

Find the other solution of the equation.

Q. 2 $y = \sin\left(\log_e x^3\right)$.

What is the value of $\frac{dy}{dx}$ when $\boldsymbol{x} = 1$?

ROUND FIVE

Q. 1 The parametric equations

$$x = \frac{8t}{1+4t^2}, \qquad y = \frac{2-8t^2}{1+4t^2}$$

define the circle

$$x^2 + y^2 = r^2.$$

Find the value of r.

Q. 2 Find the 2×2 matrix M such that

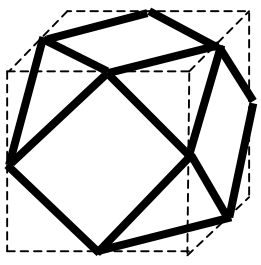
$$\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix} M \begin{pmatrix} 4 & -3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}.$$

ROUND SIX

Q. 1 A die has six square faces and eight triangular faces. When thrown, it always lands on one of its faces and is twice as likely to land on any square face as any triangular face.

When the die is thrown, what is the probability that the face it lands on is square ?

Write the answer in the form $\frac{a}{b}$, where $a, b \in \boldsymbol{N}$.



Q. 2 The die in question 1 is made by cutting away the eight corners through the midpoints of the edges of a regular 6-faced die.
Each edge of the 6-faced die was 2 cm in length.

Find the sum of the edges of the resulting 14-faced die in the form $a\sqrt{b}$, where $a, b \in \boldsymbol{N}$.

ROUND SEVEN

Q. 1 Find the shortest distance between the lines

$$3x + 4y - 10 = 0 \quad \text{and} \quad 3x + 4y = 0.$$

Q. 2 Evaluate $\int_0^1 \frac{1}{e^{\sqrt{x}} . \sqrt{x}} \, dx$

Give your answer correct to two decimal places.

Q. 3 From a boat which is due east of a lighthouse, the angle of elevation of the top of the lighthouse is $45°$. From another boat which is due north of the lighthouse, the angle of elevation of the top of the lighthouse is $30°$.
The top of the lighthouse is 200 metres above the level of each boat.

Calculate the distance between the two boats.

Q. 4 Find a value of x that satisfies the equation

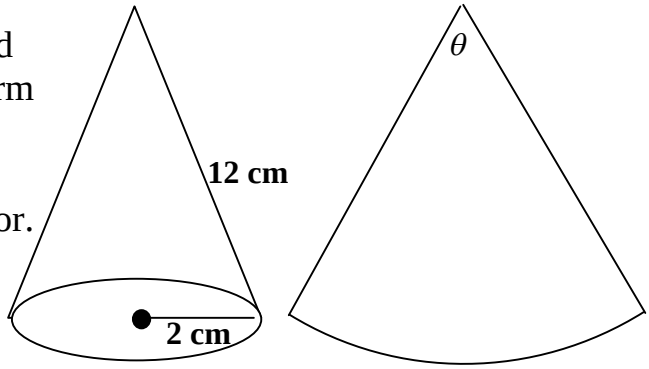
$$\log_3(4 - x) + \log_3(6 - x) = 1.$$

ROUND EIGHT

Q. 1 A hollow cone of base radius 2 cm and slant height 12 cm is opened out to form a sector of a circle.

θ is the angle at the centre of the sector.

Find, in degrees, the value of θ.



Q. 2 Write, in the form $ax + by + c = 0$ where $a, b, c \in \mathbf{Z}$, the equation of the tangent to the curve

$$x^2y^2 - x^2 + y^2 = 7 \text{ at the point } (1, 2)$$

Q. 3 Solve the equation

$$6^x = 300,$$

giving your answer correct to two decimal places.

Q. 4 In a game show, each of five contestants chooses a set of boxes of a particular colour.
No two contestants choose the same colour.
There are six boxes in each colour.
One of the thirty boxes contains a prize of a car and all of the other boxes have small cash prizes.
All of the boxes are equally likely to contain the prize of the car.

Each contestant chooses a colour and then chooses boxes, one at a time, in that colour.

What is the probability that the **third** contestant to choose a colour will win the car when she chooses her **second** box?

TIE BREAK 1

Write the answers on this page and hand it up

Q. 1 $y = \sin 5$.

Find $\frac{dy}{dx}$. **Answer** _____

Q. 2 Find $\int \frac{1}{\sqrt{x^3}} dx$.

Write your answer in surd form. **Answer** _____

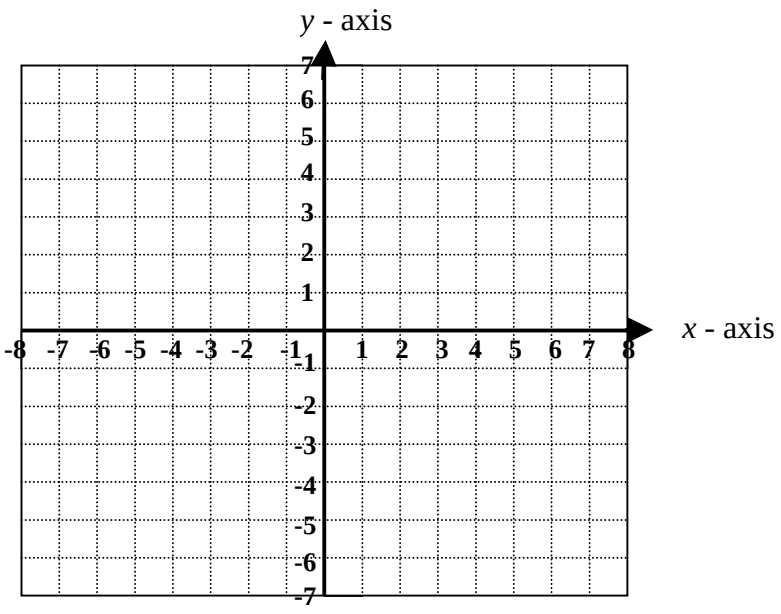
Q. 3 Find the values of $\boldsymbol{x}$ which satisfy the equation

$\cos^2 2\boldsymbol{x} - \sin 2\boldsymbol{x} = 1$, where $0^o \leq \boldsymbol{x} \leq 180^o$.

Answer _____

Q. 4 Within the limits of the given diagram, plot the graph of

$y = |x|$, where $x \in \boldsymbol{R}$.



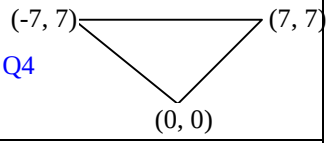
TIE BREAK 2

Write the answer on this page and hand it up

Using only digits from the set {3, 5, 6, 8}, how many numbers greater than 600 can be made, if no two digits in any number are the same?

Answer _____

ANSWERS - Round 1, 2010.

ROUND	Question 1.	Question 2.	Question 3.	Question 4.
1	Q1. 1	Q2. $\frac{1}{12}$		
2	Q1 $-\frac{1}{2}-i\frac{\sqrt{3}}{2}$	Q2 60^o and 240^o		
3	Q1. $\{x \mid x < -12\} \cup \{x \mid x > 2\}$	Q2. $4x - 1y + 20 = 0$ and $4x - 1y - 14 = 0$		
4	Q1. $2 + i$	Q2. 3		
5	Q1 $r = 2$	Q2 $\begin{pmatrix} -1 & -3 \\ -1 & -4 \end{pmatrix}$		
6	Q1. $\frac{3}{5}$ or equivalent	Q2. $24\sqrt{2}$ cm		
7	Q1 2	Q2 1·26	Q3. 400 metres	Q4. $x = 3$
8	Q1 60^o	Q2. $3x + 4y - 11 = 0$	Q3. $x = 3{\cdot}18$	Q4 $\frac{1}{30}$ (*or* $0{\cdot}0\dot{3}$ *or* equivalent)
TIE 1	Q1 0	Q2 $-\frac{2}{\sqrt{x}} + c$	Q3 0°, 90°, 135°, 180°	Q4 
TIE 2	Q1 36			