

ROUND ONE

IMTA 2008

1. Given that $0 \leq A \leq 90^\circ$ and that $\cos A = \frac{3}{5}$, find the value of $\frac{1}{\sin A}$.

Answer in the form $\frac{a}{b}$ where a and $b \in N$.

2. For what real values of x is $f(x) = \frac{x}{1-x^2}$ not defined?

ROUND TWO

IMTA 2008

1. The line with the equation $y = ax + b$ passes through the points $(-1, 0)$ and $(0, -2)$. Find the value of a and the value of b .

2. Find the equation of the tangent to the circle with the equation

$$(x-3)^2 + (y+2)^2 = 5 \text{ at the point } (2, -4).$$

Give your answer in the form $ax + by + c = 0$, where a, b and $c \in N$.

ROUND THREE

IMTA 2008

1. Find the value of A and the value of B where

$$\frac{5}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}.$$

2. X is the matrix $\begin{pmatrix} -2 & 4 \end{pmatrix}$ and Y is the matrix $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Calculate the product XY.

ROUND FOUR

IMTA 2008

1. Express $\frac{8-i}{2+i}$ in the form $a+bi$ where a and $b \in \mathbb{Z}$ and $i = \sqrt{-1}$.

2. Given that $2\sin^2 x + \cos^2 x = \frac{25}{16}$ and that x is an angle in the first quadrant find the value of $\sin x$.

ROUND FIVE

IMTA 2008

1. In a TV debate on the health service, 6 people are chosen at random from a panel which consists of 5 nurses, 3 administrators and 4 consultants.

What is the probability that no consultant is chosen?

2. Find the value of x for which

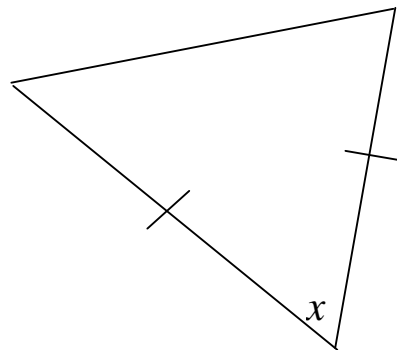
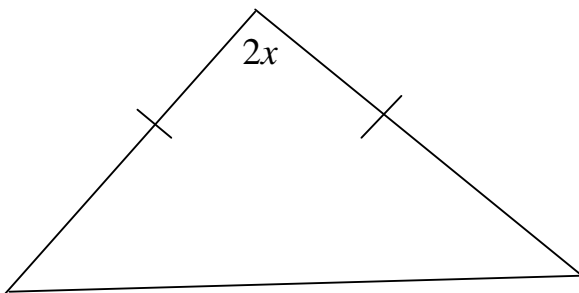
$$2 + \sqrt{x-2} = x-2, x \in R.$$

ROUND SIX

IMTA 2008

1. Evaluate $\frac{1}{\pi^2} \int_0^3 \frac{\tan^{-1} \frac{x}{3}}{x^2 + 9} dx.$

2.



The two triangles are equal in area and the marked sides are equal in length.

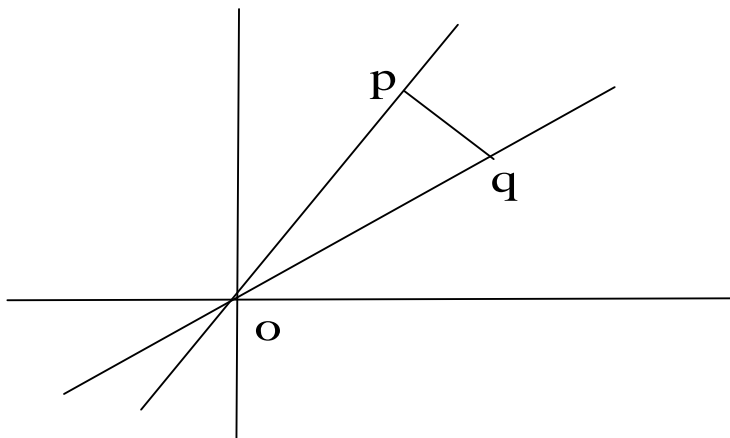
Calculate the value of x .

ROUND SEVEN IMTA 2008

1. Given $y = \sqrt{\frac{e^x}{1+e^x}}$ find the value of $\frac{dy}{dx}$ for $x = 0$.

Answer in the form $\frac{1}{a\sqrt{b}}$, where $a, b \in \mathbb{N}$.

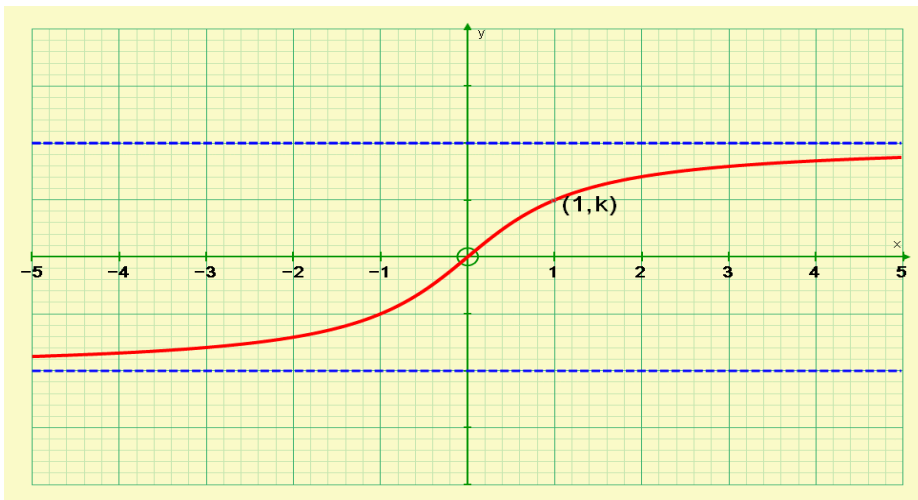
2. A bag contains 5 red markers and n blue markers. One marker is drawn and not replaced. A second marker is then drawn. If the probability that both are blue is one sixth how many blue markers are in the bag?
3. The slopes of the two lines oq and op are 1 and 2 respectively. Given that $|op| = 1$, and that $pq \perp op$, calculate the length of pq .



4. Find the equation of the tangent to the curve $x^2y + xy^2 = 6$ at the point $(2,1)$.
Give your answer in the form $ax + by + c = 0$, where a, b and $c \in \mathbb{Z}$.

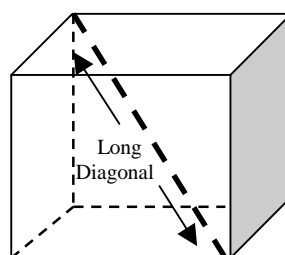
ROUND EIGHT IMTA 2008

1. Find the real numbers p and q such that $(p + iq)^2 = 15 - 8i$.
2. A section of the graph of $y = \tan^{-1} x$ is shown.



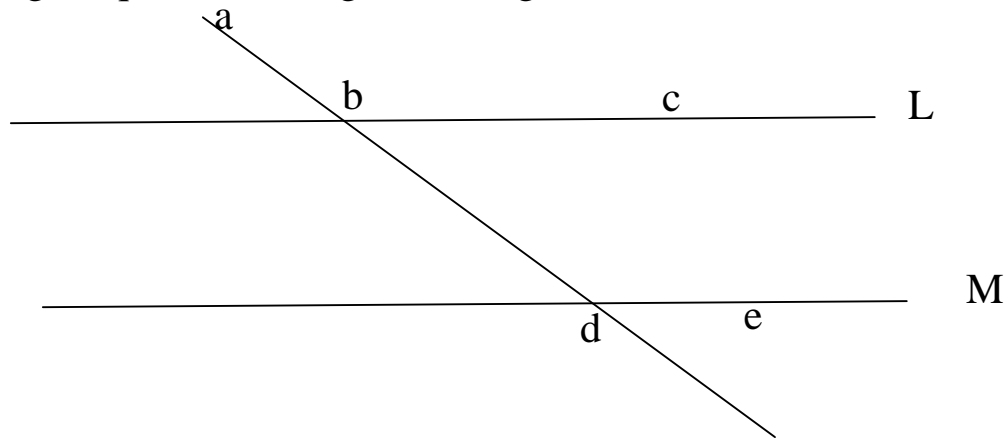
What is the value of k ?

3. Solve $2e^{2y} - 3e^y - 2 = 0$.
4. The volume of a solid cube in cm^3 plus three times the total length of its edges in cm is equal to twice its surface area in cm^2 . Calculate the length of its long diagonal in cm . Answer in the form $a\sqrt{b}$, where a and $b \in \mathbb{N}$.



TIE BREAK IMTA 2008

1. Arrange $7, 2\sqrt{11}, 4\sqrt{3}, 5\sqrt{2}, 3\sqrt{5}$. in order, putting smallest number first.
2. Find the slope of the line joining $(3, 5)$ and $(-1, 2)$ in the form $\frac{a}{b}$, $(a, b \in N)$.
3. $y = \cos x \Rightarrow \frac{dy}{dx} =$
4. $3x = 2y$ and $xy = 24$. Find a value for x and a value for y .
5. $\frac{(n+1)!}{n!} =$
6. Name an angle equal to the angle $\angle abc$ given that $L \parallel M$.



7. Factorise fully $4x^2 - 16y^2$.
8. $(a - b)^3 =$
9. $\frac{10!}{3!7!} =$
10. $3 + 4 \times 5 =$

ANSWERS - Round 1, 2008.

ROUND	Question 1.	Question 2.	Question 3.	Question 4.
1	Q1. $\frac{5}{4}$	Q2. $x = \pm 1$		
2	Q1. $a = b = -2$	Q2. $x + 2y + 6 = 0$		
3	Q1. $A = 5$ $B = -5$	Q2. 2		
4	Q1. $3 - 2i$	Q2. $\frac{3}{4}$		
5	Q1. $\frac{1}{33}$	Q2. $x = 6$		
6	Q1. $\frac{1}{96}$	Q2. $x = 60$		
7	Q1. $\frac{1}{4\sqrt{2}}$	Q2. 4	Q3. $\frac{1}{3}$	Q4. $5x + 8y - 18 = 0$
8	Q1. $(4, -1), (-4, 1)$	Q2. $\frac{\pi}{4}$	Q3. $\ln 2$	Q4. $6\sqrt{3}$

Tie-break – R1, 2008

Question	Answer
1	$2\sqrt{11}, 3\sqrt{5}, 4\sqrt{3}, 7, 5\sqrt{2}$
2	$\frac{3}{4}$
3	$-\sin x$
4	$x = \pm 4, y = \pm 6.$
5	$n + 1$
6	$\angle bde$
7	$4(x - 2y)(x + 2y)$
8	$a^3 - 3a^2b + 3ab^2 - b^3$
9	120
10	23