

Cumann Oidí Matamaitice na hÉireann
Ollscoil Luimnigh
Márta 3, 2012

– Irish Mathematics Teachers Association
University of Limerick
March 3, 2012



UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH



Irish Maths Teachers Association

Team Math Final – 2012

University of Limerick

Saturday March 3rd, 2012



Foireann Mata 2012 (An Chraobh) : Team Maths 2012 (Final)

BABHTA 1 - ROUND 1

- 1) Find the shortest distance between the circles

$$x^2 + y^2 = 9 \text{ and } x^2 + y^2 - 12x + 6y + 41 = 0.$$

Give your answer in surd form.

- 2) Find the numerical value of $2^{\log_3 3^5}$.



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BABHTA 2 - ROUND 2

- 1) The houses along the street in which John lives are numbered using consecutive odd numbers, starting with 1 on one side of the street, and by using even numbers on the other side.

John's house number is 137. If the numbering had commenced at the other end of the street John's house would have been number 85.

How many houses are there on John's side of the street?

- 2) Find the range of values of x for which

$$\frac{x+1}{x-1} \geq 2$$



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BABHTA 3 - ROUND 3

- 1) If 3 unbiased coins are tossed 8 times, find the probability of getting 2 heads and 1 tail exactly 3 times.

Answer correct to 3 significant figures.

- 2) Find the equations of the lines which pass through the point (2,1) and are inclined at an angle of 45° to the line $y = 3x + 3$.

Answers in the form $ax+by+c = 0$, where a, b and $c \in \mathbb{Z}$.



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BABHTA 4 - ROUND 4

1) Solve $\frac{x}{4} + \frac{y}{3} = \frac{9}{2}$

$$\frac{4}{x} + \frac{3}{y} = 1$$

Write your answers in coordinate form (x, y) .

- 2) There are 18 points in a plane. Five of the points are in a straight line. Apart from these, no three of the points are collinear. How many triangles can be formed by joining the 18 points?



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BABHTA 5 - ROUND 5

- 1) A six-seater carriage has two rows of three seats opposite each other.

Four people enter this carriage. If one couple must sit beside each other, how many arrangements are possible?

- 2) If x is a positive number and $x \neq 1$ express

$$\frac{1}{\log_3(x)} + \frac{1}{\log_4(x)} + \frac{1}{\log_5(x)} \text{ as a single log.}$$



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BABHTA 6 - ROUND 6

1) If $\sin(\theta + \alpha) - \cos(\theta + \alpha) = \cos(\theta)\sin(\alpha)$, calculate the value of $\tan(\theta)$,
given that $\tan(\alpha) = \frac{7}{24}$.

2) A teacher bought 100 puzzles for €82.90 to give out to members of her maths team. She bought three different types of puzzle.

Type A cost 40c each, type B cost 70c each and type C cost €1 each.

How many more type C puzzles than type A puzzles did she buy?



Foireann Mata 2012 (An Chraobh) : Team Maths 2012 (Final)

BABHTA 7 - ROUND 7

- 1) When Mary and Julia ran a 100 metre race, Mary won by 10 metres.
When Julia ran against Anne, Julia won that race by 20 metres.
Assuming that Mary and Anne race at exactly the same speeds as each ran in her previous race, by how much would Mary beat Anne?

- 2) If crocodiles consider that:
one dog is worth three rats, one pig is worth as much as a dog and a rat, and five cats are worth three dogs, then how many cats would ninety seven rats and thirty two pigs be worth?

- 3) The expression $ax^2 + bx + c$ takes the value 2 when $x = -2$ and when $x = 4$ and its minimum value is -7 .
Find the value of
$$a + b + c.$$

- 4) A triangle has an area of 9 square units. It is formed by the x -axis and the lines $x = 1$ and $y = mx - 4$

If $m < 0$, find all possible values of m .



Foireann Mata 2012 (An Chraobh) : Team Maths 2012 (Final)

BABHTA 8 - ROUND 8

- 1) Find the coordinates of P, the point of intersection of the curves
 $y = e^x$ and $y = 2 + 3e^{-x}$

Answer in non-decimal format.

- 2) Find the value of $\int_0^{\pi} \sin^2(x)\cos^2(x)dx$

Answer in terms of π .

- 3) Write down the set of all the positive integer solutions to the equation

$$a^2 - 7a + b^2 - 7b + 2ab = 0$$

Answers in coordinate form (a, b) .

- 4) A computer is programmed to scan the digits of the counting numbers, starting at 1.

For example, if it scans

1 2 3 4 5 6 7 8 9 10 11 12, it has scanned 15 digits.

The computer begins its task and scans the first 1788 digits.
What was the last counting number scanned?



Answers (Final 2012)

Round 1	Round 2
(1) $3\sqrt{5} - 5$ (2) 32 or 2^5	(1) 111 (2) $1 < x \leq 3$
Round 3	Round 4
(1) $\cdot 282$ or 0.282 to 3 significant figs (2) $2x + y - 5 = 0$ and $x - 2y = 0$	(1) (6, 9) and (12, 4.5) (2) 806
Round 5	Round 6
(1) 96 (2) $\log_x(60)$	(1) $\frac{24}{31}$ (2) 43
Round 7	Round 8
(1) 28 m (2) 125 cats (3) -7 (4) $m = -2$ or -8	(1) $(\ln(3), 3)$ (2) $\frac{\pi}{16}$ (3) (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) (4) 632



Scoilt : Tiebreak – Final 2012

- 1) If $f(x) = x^2 - 7x + k$ and $f(k) = -9$, find the value of $f(-1)$.
- 2) Find the value of x if $\log_3(\log_2(x)) = 2$.
- 3) If $\cos(A) = \frac{3}{5}$ and $\pi \leq A \leq 2\pi$, find the value of $\tan(A)$.
Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$.
- 4) Find the length of the longest possible chord of the circle $x^2 + y^2 = 16$ that passes through $(1, -2)$.
- 5) A palindrome is a number that remains the same when its digits are written in reverse order. For example 252 is a palindrome.
A car's clock-distance (in kilometres) reads 15951. Find the least number of kilometers required for the next palindrome to appear.
- 6) An unbiased coin is tossed four times. On the first three occasions it turns up tails. What is the probability that on the fourth occasion it will again turn up tails?
- 7) The roots of the equation $x^2 + x + 2 = 0$ are α and β .
Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$.
- 8) A committee consists of 5 elected members. The members are to be elected from 5 women and 3 men. Find the number of ways the committee may be elected when not more than 2 men may be on the committee.
- 9) Find the 9th term of the expansion $(x - \frac{1}{\sqrt{x}})^{12}$ in simplest form.
- 10) Find the area of the triangle with vertices $(0, 0)$, $(-1, 3)$ and $(7, -1)$
- 11) If repetitions are not allowed, how many three-digit odd numbers can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9? A three-digit number cannot begin with 0.
- 12) Find the numerical value of $\int_3^7 \frac{xdx}{1+x^2}$ as a single term.
- 13) Solve for x : $3x^2 + 2x + 2 < 2x^2 + x + 4$.
- 14) Two numbers have a sum of 8 and a product of 15.
What is the sum of the reciprocals of these numbers?



Tiebreak answers (Final 2012)

1)	11	8)	46
2)	512	9)	495
3)	$-\frac{4}{3}$	10)	10
4)	8	11)	320
5)	110	12)	$\ln \sqrt{5}$ or $\frac{1}{2} \ln(5)$
6)	$\frac{1}{2}$	13)	$-2 < x < 1$
7)	$-\frac{1}{2}$	14)	$\frac{8}{15}$