OLLSCOIL LUIMNIGH

UNIVERSITY OF LIMERICK

TEAM MATH FINAL 2011

ROUND 1

Time allowed: **<u>Six</u>** minutes

Q. 1 Solve the simultaneous equations

$$\begin{aligned} 2x + y + z &= 0 \\ 3x - 2y + 6z &= -6 \\ 3y + 2z &= 11. \end{aligned}$$

Q. 2 Calculate the distance from the point $(-2, 3)$ to the line

$$5x - 12y - 6 = 0.$$

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ROUND 2

Time allowed: **Six** minutes

Q. 1 Write, in the form $a + bi$, the values of z that satisfy the equation $z^2 + i = 0$, where $a, b \in \boldsymbol{R}$.

Q. 2 Find the centre and radius of the circle

$$2x^2 + 2y^2 + 6x - 8y = 0.$$

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ROUND 3

Time allowed: <u>**Six**</u> minutes

Q. 1 Find the equations of the two circles that contain the points (–2, 3) and (4, 1) and have radius 10.
Write your answers in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.

Q. 2 Find the range of values of x for which

$$\frac{x -}{x} > \quad , \ x \neq 0.$$

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ROUND 4

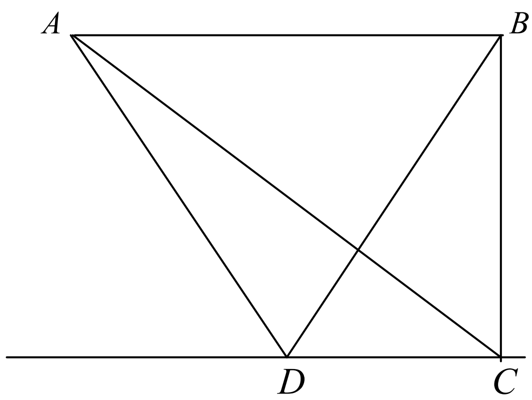
Time allowed: **Six** minutes

Q. 1 Solve the equation

$$\log_e(2x-5)+\log_e(x-3)=2\log_e(2x-6)-\log_e 2.$$

Q. 2 In the diagram,
$|\angle CBA| = 90^\circ$, $|\angle ACB| = 20^\circ$,
$|BC| = 20$
$DC \mathbin{/\!/} AB$ and $|AD| = |BD|$.

Calculate $|\angle ADB|$ in degrees, correct to one decimal place.



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ROUND 5

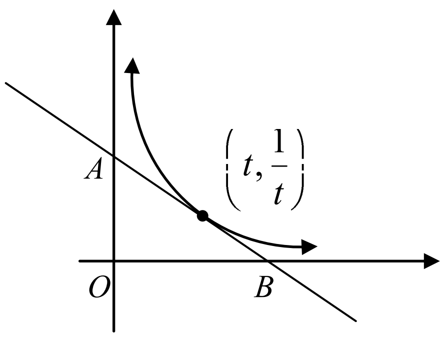
Time allowed: **Six** minutes

Q. 1 Evaluate

$$\int_0^1 \frac{2x+1}{1+x^2}\,dx\,.$$

Write your answer in terms of π and log.

Q. 2 A tangent to the curve $y = \dfrac{1}{x}$ at the point $\left(t, \dfrac{1}{t}\right)$ intersects the y-axis at A and the x–axis at B.



Find the co-ordinates of A and B in terms of t.
Find the area of triangle AOB, where O is the origin.

Team Number ________

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ROUND 6

Time allowed: **Six** minutes

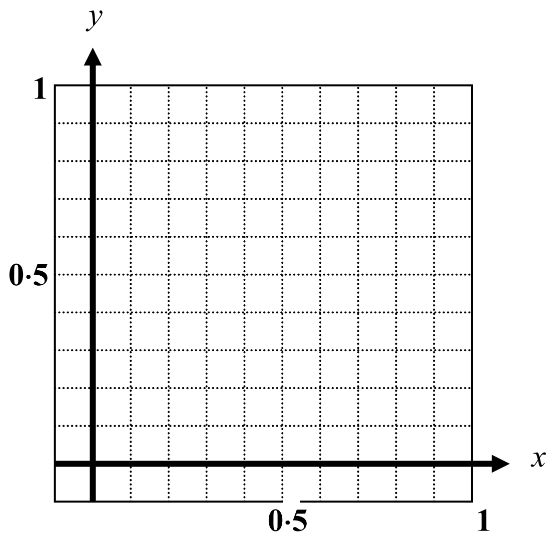
Please answer the questions on this question paper and hand it up.

Q. 1 Within the limits of the given diagram, clearly indicate the set of points that satisfy

$$x \geq 2y,$$

where $x, y \in \boldsymbol{R}$,

$x \geq 0$ and $y \geq 0$.



Q. 2 Express tan $22{\cdot}5^{\circ}$ in its simplest surd form.

Answer to question 2 ______________________________

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ROUND 7

Time allowed: **Six** minutes

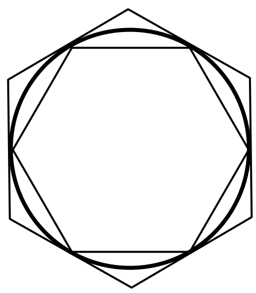
Q. 1 $x = \cos^2 t$, $y = \cos 2t$.

Find $\dfrac{dy}{dx}$ in its simplest form.

Q. 2 Points A and B are on a given plane and are 6 units apart.
How many lines can be drawn on that plane so that A is 2 units from each of the lines and B is 4 units from each of the lines?

Q. 3 Find the total number of ways in which six people can be divided into two groups with at least one person in each group and where all six people are involved each time.

Q. 4 A regular hexagon is inscribed in a circle (radius r) and another regular hexagon is circumscribed about the circle.



Find the ratio of the area of the inscribed hexagon to that of the circumscribed hexagon.

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ROUND 8

Time allowed: **<u>Six</u>** minutes

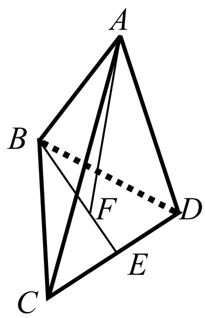
Q. 1 Find the values of x that satisfy the equation

$$\cos x + \cos 3x + 2\cos 2x = 0, \text{ where } 0 \leq x \leq 180^{\circ}.$$

Q.2 For what value of x is

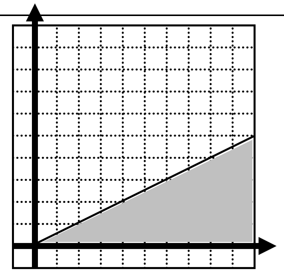
$$9^{x}.4^{x} = 216?$$

Q. 3 $ABCD$ is a regular tetrahedron whose edges are of length 6.
$|CE| = |ED|$
$AF \perp BE$ and $|BF| : |FE| = 2 : 1$.
Calculate $|AF|$.
Write your answer in surd form.



Q. 4 In a talent show, five judges, including Dana and John are randomly seated in a row.
What is the probability that Dana is seated to the left of John but not next to him?

FINAL, 2011
ANSWERS

Round	Question 1	Question 2	Question 3	Question 4
1	$x = -2, y = 3, z = 1$	Distance = 4		
2	$z = \mp \frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}} i$	$(-1{\cdot}5, 2)$, $r = 2{\cdot}5$		
3	$x^2 + y^2 + 4x + 14y - 47 = 0$ $x^2 + y^2 - 8x - 22y + 37 = 0$	$x < 0$		
4		$\lvert \angle ADB \rvert = 20{\cdot}6^{\circ}$		
5	$\frac{\pi}{4} + \mathrm{og}_e 2$	$A(0, \frac{2}{t}), B(2t, 0)$ Area = 2		
6		$\tan 22{\cdot}5^{\circ} = \sqrt{2} - 1$		
7	$\frac{dy}{dx} = 2$	3 lines	31 ways	3 : 4
8	$x = 45^{\circ}, 135^{\circ}, 180^{\circ}$	$x = 1{\cdot}5$	$\lvert AF \rvert = \sqrt{24}$ or $2\sqrt{6}$	$\frac{3}{10}$