OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

TEAM MATH FINAL 2009

ROUND 1

1. The lengths of the 3 sides of a right-angled triangle are given by

$$a-d, \quad a, \quad \text{and} \quad a+d$$

where d is a positive number.

Find the value of

$$\frac{a}{d}.$$

2. Find all the values of k satisfying the equations

$$k\cos\alpha = 4 \quad \text{and} \quad k\sin\alpha = -3.$$

ROUND 2

3. Find the equation of the tangent line to the circle

$$(x-1)^2 + (y-2)^2 = 13,$$

at the point (3, 5).

Give your answer in the form $y = m\,x + c$.

4. Let A and B be the matrices given by

$$A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4 & 1 \\ 1 & -1 \end{pmatrix}.$$

Express the matrix $B^{-1}\,A\,B$ in the form $\begin{pmatrix} m & 0 \\ 0 & n \end{pmatrix}$, where m and n are natural numbers.

ROUND 3

CALCULATORS MAY NOT BE USED IN THIS ROUND

5. Find the exact value of
$$\sqrt{\frac{8^{10}+4^{10}}{8^4+4^{11}}}.$$

6. For all real numbers x, except $x = 0$ and $x = 1$, the function f is given by
$$f\left(\frac{x}{x-1}\right) = \frac{1}{x}.$$
If $0 < \theta < \frac{\pi}{2}$, evaluate $f((\sec\theta)^2)$.

ROUND 4

7. Find all the values of x which satisfy the equation
$$e^{2x} - 3e^x - 10 = 0.$$

8. Find the equation of the tangent line to the circle
$$x = 3\cos t \quad \text{and} \quad y = 3\sin t$$
at the point $(0, -3)$.
Give your answer in its simplest form.

ROUND 5

9. The quadratic equations
$$x^2 + x - 1 = 0 \quad \text{and} \quad x^2 - x + d = 0$$
have a common root. Find all possible values for d.
Give your answers in the form $a + \sqrt{b}$ where $a,\ b \in \mathbf{Z}$.

10. Evaluate
$$\int (\cos x + \sin x)^2\, dx.$$

ROUND 6

11. Find all solutions to the equation

$$z^2 = 3 + 4\,i.$$

Give your answers in the form $a + b\,i$, where $a,\ b \in \mathbf{R}$.

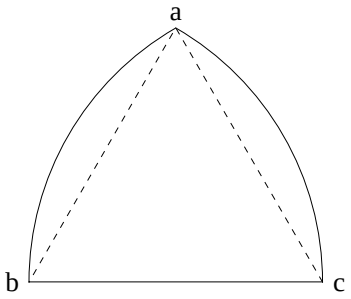
12. Simplify

$$(1+i)^9 - (1-i)^9.$$

Give your answer in the form $a + i\,b$ where $a,\ b \in \mathbf{Z}$.

ROUND 7

13. In the diagram, the points **a**, **b**, and **c** are the vertices of an equilateral triangle with side lengths $2\sqrt{3}$ cm. The point **b** is the centre of the arc **ac** and the point **c** is the centre of the arc **ab**. Find the area of the figure bounded by the two arcs **ab** and **ac** and the line segment **bc**.



Give your answer in the simplest $m\pi + n\sqrt{p}$ form, where m, n and $p \in \mathbf{Z}$..

14. A point is picked at random on the circumference of the circle whose equation is $x^2 + y^2 = 1$. The selected point is then joined to the centre of the circle. What is the probability that the slope of this line segment is less that $\frac{1}{\sqrt{3}}$?

Give your answer in the simplest $\frac{m}{n}$ form, where m and n are natural numbers.

15. The point $(p,\ k)$ lies on the curve whose equation is $y = \log_e x$ and the point $(q,\ k)$ lies of the curve whose equation is $y = \frac{1}{2}\log_e x$. Find the value of q if $p = 5$.

16. Given that $0 \leq x \leq \frac{\pi}{2}$, find all solutions to the equation

$$\frac{3 + \sin^2 3\,x}{\cos 3\,x - 2} = 3\,\cos 3\,x.$$

Give your answers in the simplest $(\frac{m}{n})\pi$ form, where m and n are natural numbers.

ROUND 8

17. Let 1, ω and ω^2 denote the roots of $z^3 = 1$.

    Evaluate $(1 - \omega - \omega^2)^5$.

18. Let $p(x) = a\,x^2 + b\,x - 6$. If $p(x)$ is divided by $x - 2$ the remainder is 4 and if $p(x)$ is divided by $x + 1$ the remainder is -5.

    Find the roots of the equation $p(x) = 0$.

19. A triangle has vertices **a**, **b** and **c**. The lengths of the sides **ab** and **ac** are 1.0 cm. and 2.5 cm, respectively. If $\tan(\widehat{\mathbf{bac}}) = 0.75$, find the area of the triangle.

    Give your answer in decimal form.

20. Three fair dice are rolled. What is the probability that the sum of the three numbers appearing on top is 15.

    Give your answer in the simplest form $\frac{m}{n}$, where m and n are natural numbers.

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TEAM MATH FINAL 2009

TIE BREAK ROUND

Time allowed: **Six** minutes

1. If (7, 7) is the midpoint of the line segment joining the points (1, 4) and (a, 10), what is the value of a?

2. Two *different* numbers are chosen at random from the set

$$\{0, 1, 2, 3, 4\}.$$

What is the probability that the sum of the two selected numbers is greater that their product?

Give your answer in the simplest $\frac{m}{n}$ form, where m and n are natural numbers

3. Find all the solutions for x in the equation

$$\log_3(x+4) + \log_3(6-x) = 2.$$

4. Solve for x in the equation

$$8^{x-1} = \left(\frac{1}{16}\right)^{5-x}.$$

ANSWERS 2009				
Round	Question 1	Question 2	Question 3	Question 4
1	4	± 5		
2	$y = -\frac{2}{3}x + 7$	$\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}$		
3	16	$\sin^2\theta$		
4	ln 5	$y = -3$		
5	$-2 \pm \sqrt{5}$	$x - \frac{1}{2}\cos 2x + c$		
6	$2+i, \quad -2-i$	$32i$		
7	$\frac{3\sqrt{3}}{\pi}$	$\frac{2}{3}$	5	$40°(\frac{2\pi}{9})$ and $80°(\frac{4\pi}{9})$
8	32	$\left\{\frac{3}{2}, -\frac{2}{1}\right\}$	$0 \bullet 75\ \text{cm}^2$	$\frac{5}{108}$