

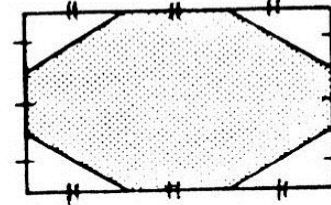


Babhta 1

Round 1

- 1) The sides of a 12 x 9 rectangle are trisected.

The points are joined as shown in the diagram.
Calculate the perimeter of the octagon formed.



- 2) Given the following pattern

$$\begin{aligned} 1 + 2 &= 3 \\ 4 + 5 + 6 &= 7 + 8 \\ 9 + 10 + 11 + 12 &= 13 + 14 + 15 \\ 16 + 17 + 18 + 19 + 20 &= 21 + 22 + 23 + 24 \\ \dots \end{aligned}$$

If the pattern continues, find the value of the first term immediately after the “=” in the 20th row.

Babhta 2

Round 2

- 1) How many 5-digit positive integers contain the digit 6 exactly once?
A 5-digit number is $\leq 10\,000$.
- 2) What is the smallest positive integer n such that $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^n = 1$?

Babhta 3

Round 3

- 1) Find the value of the integral $\int_0^4 e^{2x} dx$ in terms of e .
- 2) Solve for x : $\sqrt{16^{x+1}} = 8^{1-x}$, where $x \in \mathbb{Q}$.

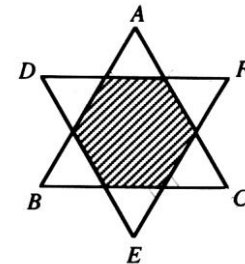
Answer in the form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

Babhta 4**Round 4**

- Two sides of a parallelogram lie along the lines $x + y = 5$ and $3x + y = 9$ respectively. The diagonals of the parallelogram intersect at the point $(3, -1)$. Calculate the coordinates of all the vertices of the parallelogram.
- ABC is a right angled triangle. The right angle is at B. The line segment [AD] is the bisector of the acute angle A, where $D \perp [BC]$. [AD] divides the opposite side [BC] into segments of lengths 1 and 2, where $|BD| = 1$ and $|DC| = 2$. Calculate the length of the bisector [AD].

Babhta 5**Round 5**

- ABC and DEF are equilateral triangles, each of area 12. They overlap to form a regular hexagon. Calculate the area of the hexagon.
- The function $f(x) = ax + b$ satisfies the equalities :



$$f(f(f(1))) = 29 \text{ and } f(f(f(0))) = 2.$$

$$\text{That is } f_0 f_0 f(1) = 29 \text{ and } f_0 f_0 f(0) = 2$$

Calculate the value of a.



Babhta 6

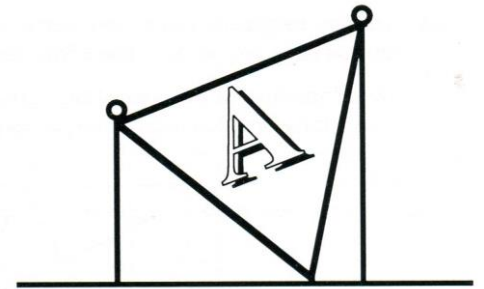
Round 6

- 1) Three six-sided dice are rolled at once.
What is the probability that at least two of the dice show the same number?

Answer in simplest form $\frac{a}{b}$, where a and $b \in \mathbb{N}$.

- 2) A large flag in the shape of an equilateral triangle is suspended by two of its corners from the tops of two vertical poles, one 4 m high and the other 3 m high. The third corner of the flag just touches the ground. Find the length of a side of the flag.

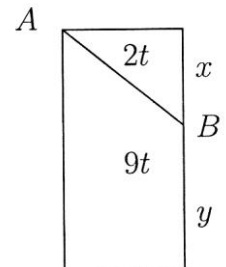
Answer in the form $\sqrt{\frac{a}{b}}$, where a and $b \in \mathbb{N}$.



Babhta 7

Round 7

- 1) The line segment [AB] divides the rectangle into two parts. The ratio of the areas of these parts is 2 : 9. Calculate the ratio of the lengths x and y in simplest form. Answer in form simplest $x : y$, where x and $y \in \mathbb{N}$.
- 2) Two machines move at constant speeds around a circle of circumference 600 cm, starting together at the same point. If they travel in the same direction then they next meet after 20 seconds. If they travel in opposite directions then they next meet after 5 seconds. At what speed, in cms, per second, is the slower machine travelling?
- 3) The Fibonacci sequence is 1, 1, 2, 3, 5, 8, Every term after the first two terms is the sum of the two previous terms. Let \mathbf{a} , \mathbf{b} , \mathbf{c} and \mathbf{d} be four successive terms of the Fibonacci sequence. If $\mathbf{a} + \mathbf{b} + \mathbf{c} = 3194$ and $\mathbf{b} + \mathbf{c} + \mathbf{d} = 5168$, find the value of \mathbf{a} .

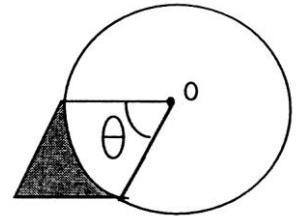




- 4) John and Mary bought a house on March 1st, 1996. They obtained a 30 year loan from the bank at an annual interest rate of 7.5% APR. Their monthly repayment at the end of each month was €1100. The first repayment was made at the end of March 1996. In February 2016 they had a substantial win in the Lotto. They paid the February 2016 instalment as usual and later decided to pay off the loan from March 2016.
How much did they have to pay to clear the loan?
Answer to the nearest euro.

Babhta 8**Round 8**

- 1) The diagram shows a disc of area 1 cm^2 , centre o , and a parallelogram whose sides are all of equal length. q is an angle in radian measure. Find an expression for the shaded area in terms of π and q .



- 2) The lines $y = mx + 1$, where m is positive integer, and $13x + 9y = 183$ intersect at a point P . For how many values of m are the coordinates of P integers?
- 3) In a series of 5 games to be played between two equally matched teams, the first team to win 3 games becomes the champion. Team X has won the first game. If there are no draws find the probability that team X will be the champion.
- 4) John wants to make a secret code of five letter “words”. He makes the following rules:
- No more than two vowels or two consonants in succession
 - No “word” to start or end with two consonants

He rejects the letter A as too obvious, so he has 21 consonants and 4 vowels to choose from.

How many possible code “words” are possible?

Note : a letter may be repeated, for example **BECCU**



Answer Key for TEAM MATHS Final 16

Round 1	Q1	34	Q2	421
Round 2	Q1	29889	Q2	8
Round 3	Q1	$2e^2$	Q2	$\frac{1}{5}$
Round 4	Q1	(2,3) , (4, -5) , (5, -6) , (1,4)	Q2	2
Round 5	Q1	8	Q2	3
Round 6	Q1	$\frac{4}{9}$	Q2	$\sqrt{\frac{52}{3}}$
Round 7	Q1	$\frac{4}{7}$	Q2	45
	Q3	610	Q4	€93680
Round 8	Q1	$\frac{\sin(q)}{p} - \frac{q}{2p}$ or $\frac{2\sin(q) - q}{2p}$	Q2	1
	Q3	$\frac{11}{16}$	Q4	647472

Answer Key Tiebreak 2016

TIEBREAK

q1	0	q 2	$\frac{x}{x - 1}$	q3	4y	q4	$\frac{1}{3}$
q5	11	q6	165 km	q7	$\frac{1}{3}(\log_{10}(x)+5)$	q8	1 + i
q9	$k = \frac{80}{19}$	q10	6	q11	$\frac{4}{3}$	q12	126



q13 2 q14 $\frac{4\sqrt{3}}{7}$

Foireann Mata 2016 (An Chraobh)**Team Maths 2016 (Final)****Scoilt****Tiebreak**

- 1) In how many different ways can fourteen 10-cent coins be divided into three piles with an odd number of 10-cent coins in each pile?
- 2 $x = 1 + 2^p$ and $y = 1 + 2^{-p}$. Write y in terms of x .
- 3) $y^2 = y + 4$.
Calculate the value of $y^4 - y^3 - 16$ in terms of y^k , where $k = 1$.
- 4) x , y and z form a geometric sequence with a common ratio r and $x^{-1}y$.
If x , $2y$ and $3z$ form an arithmetic sequence, find the numerical value of r .
Answer in simplest form $\frac{a}{b}$, where a and $b \in \mathbb{Z}$.
- 5) $f(x) = px^4 + qx^2 + x + 5$, where p and q are constants.
Find the value of $f(4)$ if $f(-4) = 3$.
- 6) If John drove at 110 km/h instead of 100 km/h he would save 9 minutes when travelling between two towns.
Calculate the distance, in km, between the towns.
- 7) Calculate the inverse of the function $f(x) = 10^{3x - 5}$.
- 8) Write $1 - i^{-1}$ in the form $a + ib$, where a and $b \in \mathbb{Z}$



9) If $\frac{\log_b(a)}{\log_c(a)} = \frac{19}{99}$, then $\frac{b}{c} = c^k$.

Find the value of k in simplest form $\frac{p}{q}$, where p and q $\in \mathbb{N}$.

- 10) There are 10 digits when using the denary system in mathematics
0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, starts with two 1's and each term after that is the sum of the two previous terms.
Which one of the ten digit listed above is the last to appear in the units position of a number in the Fibonacci sequence ?

- 11) If $4(9a - 13b) = 6(a - 2b)$, what is the ratio of a to b
in simplest form $\frac{a}{b}$, where a and b $\in \mathbb{N}$?

- 12) Each letter below represents a positive integer. A = 31 and F = 7.

31 B C D E 7 G H I

The sum of any three adjacent numbers is 164. Find the value of H

- 13) Solve for x: $\ln(x) + \ln(x+2) = \ln(x+6)$.

- 14) When $\sin(A) = \frac{1}{7}$ and $\tan(A) > 0$, calculate the numerical value of
 $\sin(A + \frac{\rho}{2})$.

Answer form $\frac{a\sqrt{b}}{c}$ where a, b and c $\in \mathbb{N}$.

Cumann Oidí Matamaitice na hÉireann :



Irish Mathematics Teachers Association

Foireann Mata 2016 (An Chraobh)

Team Maths Final 2016