Blaise Pascal, familiar to students through *Pascal's triangle*, said, when challenged on using infinitesimals:

> *"The heart has its reasons that reason does not know"*.

Formal number theory is almost unreadable.

To express

If a divides b and b divides c, then a divides c

we say

$$a \mid b \,\&\, b \mid c \supset a \mid c$$

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(Z) The two sides of this Triangle are equal to each other.

First attempt

(A) Things that are equal to the same are equal to each other.

(B) The two sides of this Triangle are things that are equal to the same.

(C) If A and B are true, Z must be true.

(Z) The two sides of this Triangle are equal to each other.

Second attempt

(A) Things that are equal to the same are equal to each other.

(B) The two sides of this Triangle are things that are equal to the same.

(C) If A and B are true, Z must be true.

(D) If A and B and C are true, Z must be true.

(Z) The two sides of this Triangle are equal to each other.

Third attempt

(A) Things that are equal to the same are equal to each other.

(B) The two sides of this Triangle are things that are equal to the same.

(C) If A and B are true, Z must be true.

(D) If A and B and C are true, Z must be true.

(E) If A and B and C and D are true, Z must be true.

(Z) The two sides of this Triangle are equal to each other.

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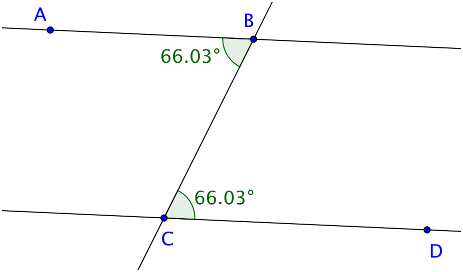
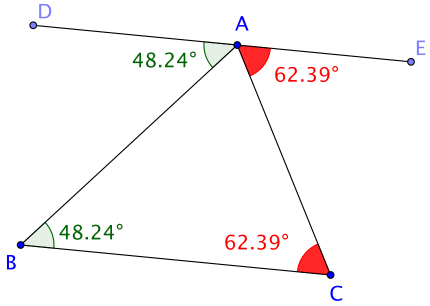
- Defined terms such as area
- Axioms such as *There is exactly one line through any two given points*
- Theorems such as *Vertically opposite angles are equal*

Experiment leads to the need for proof. Look at Theorem 3:



In Theorem 4 the teacher can show and hide lines. The student can change the triangle.



- Use surprise. Can Pythagoras still surprise your students?

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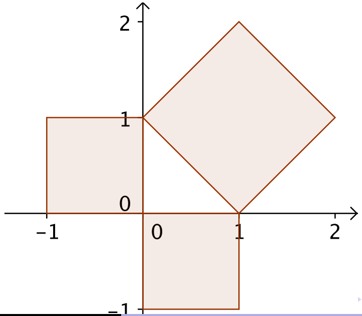
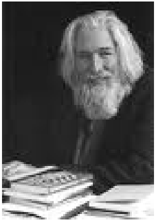
- Raymond Smullyan tells how, when introducing Pythagoras, he drew a right triangle with squares on the sides.

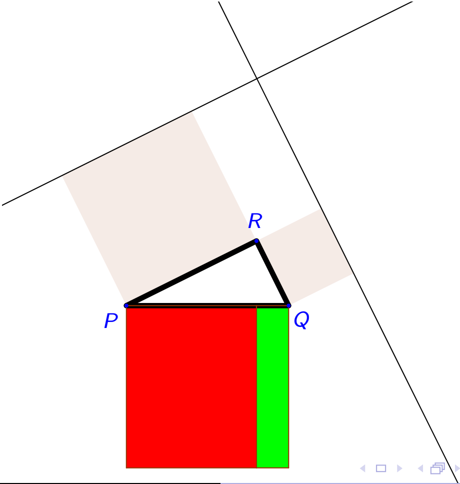




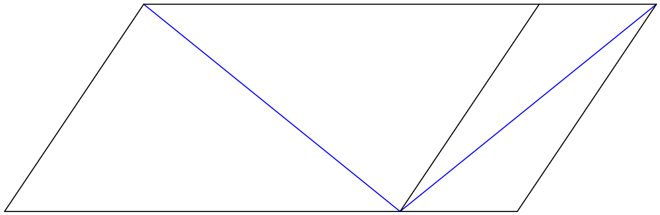
"Now suppose these three squares were made of beaten gold, and you were offered either the one large square or the two small squares. Which would you choose?".

The class divided 50-50 in what to do and both groups were equally amazed when told it would make no difference.

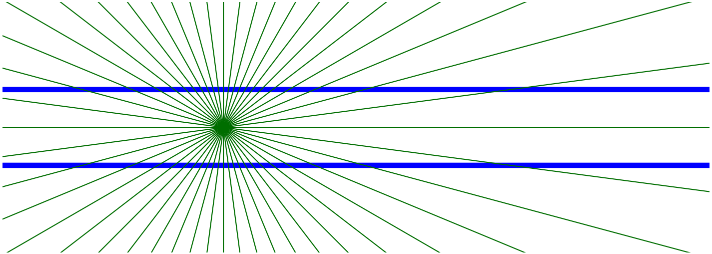
Use a demonstration to convince about the truth of Pythagoras.



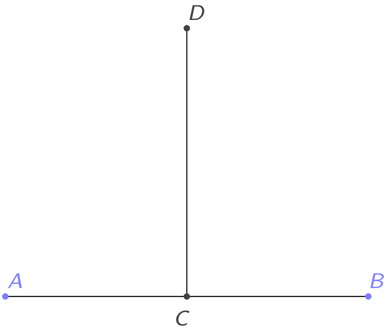
Use optical illusions.
Students love them and they emphasise the need for proof.
Which blue line is the longer?



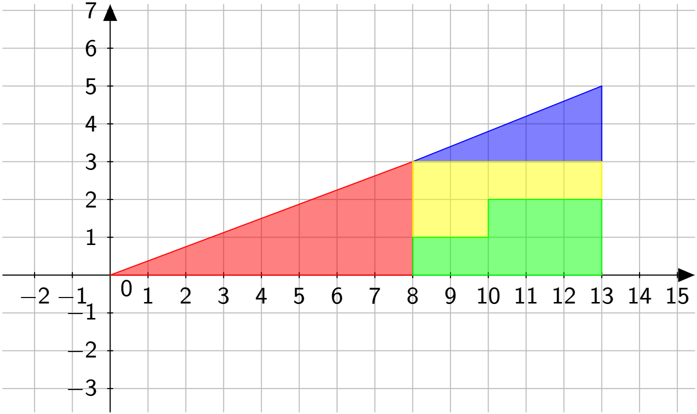
Are the blue things curves or lines?



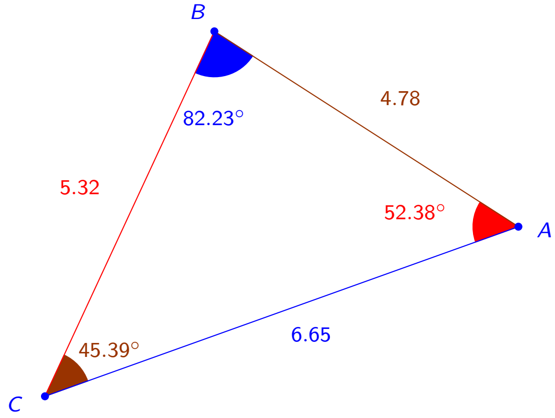
Even vertical lines give us problems!



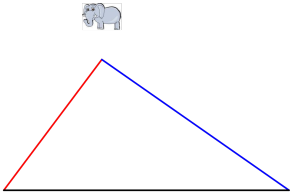
Where did the square come from?



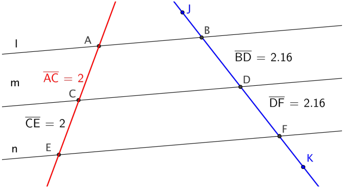
Theorem 7 (Angle opposite longer side)



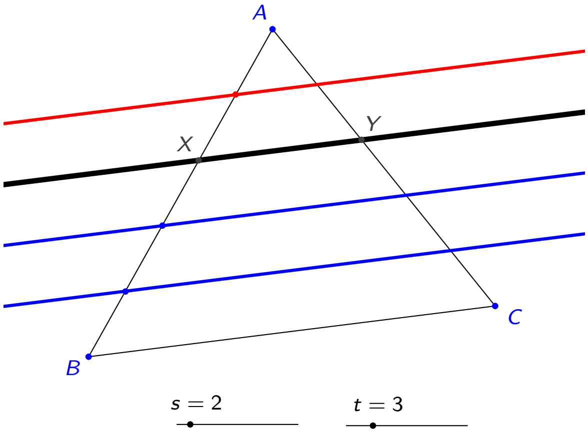
Theorem 8 (Two sides in a triangle are greater than a third)



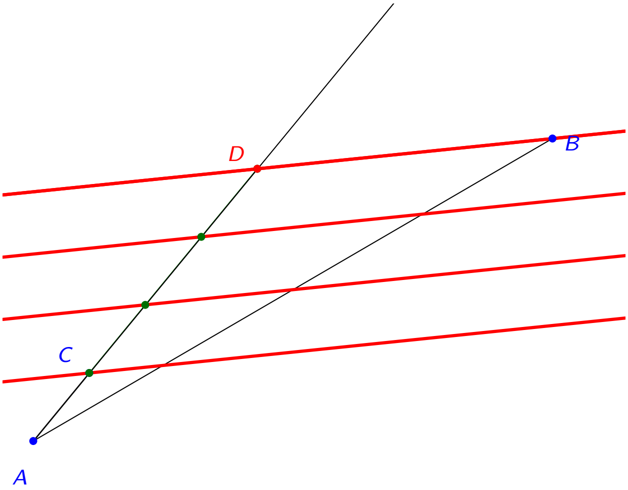
Theorem 11 (Transversals)



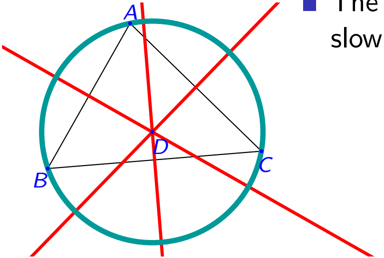
Theorem 12 (Equal Ratios)



Construction 7 (Equal divisions of a line segment)



Construction 16 (Circumcircle)



- The check-boxes reveal the steps slowly

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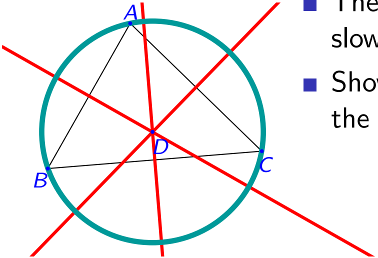

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- Show the perpendicular bisectors of the sides

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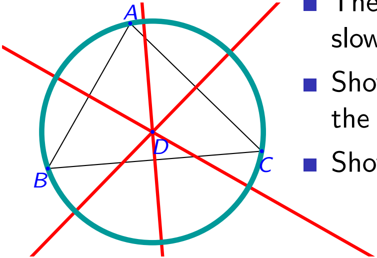

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- Show the perpendicular bisectors of the sides
- Show their intersection

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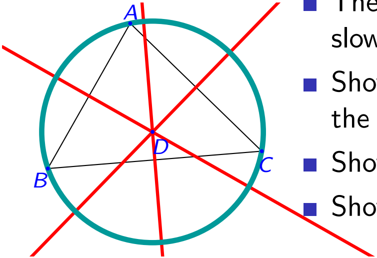

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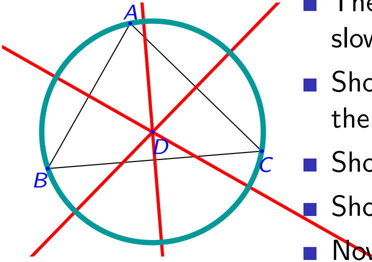
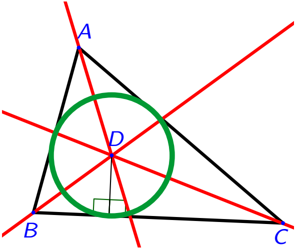


- The check-boxes reveal the steps slowly
- Show the perpendicular bisectors of the sides
- Show their intersection
- Show the circumcircle
- Now change the triangle

Construction 17 (Incircle)



- The check-boxes reveal the steps slowly
- Show the bisectors of the angles
- Show their intersection
- Show the perpendicular to a side
- Construct the incircle
- Now change the triangle

Designing an applet

How do you design an applet?

1. Pick the topic you want to demonstrate

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6. You are ready to start

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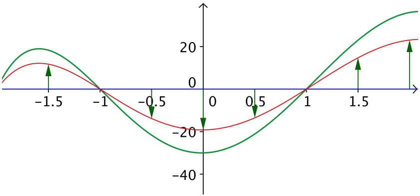
The single most useful equation is the following:
The line segment joining the point A to the point B is

$$t\,B + (1 - t)\,A$$

as t goes from 0 to 1

Designing an applet



Here I defined

$$\begin{aligned} f(x) &= (x+2)(x+1)(x-1)(x-3)(x-5) \\ h(x) &= 0 \\ g(x) &= t\,f + (1-t)\,h \end{aligned}$$

and

$$\texttt{Sequence}[\texttt{Vector}[(\texttt{k},0),(\texttt{k},\texttt{g}(\texttt{k}))],\texttt{k},-3,4,0.5]$$

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Topic Show central symmetry in action

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Steps

1. Pick central point
2. Draw figure
3. Get image
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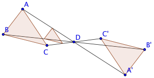

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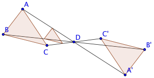
Draw 

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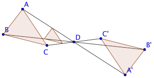

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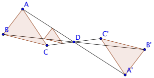
Draw 

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Begin!

Contact

My email is

timbrophy@mac.com