

# ROUND ONE

IMTA 2008

1. Given that  $0 \leq A \leq 90^\circ$  and that  $\cos A = \frac{3}{5}$ , find the value of  $\frac{1}{\sin A}$ .

Answer in the form  $\frac{a}{b}$  where  $a$  and  $b \in N$ .

2. For what real values of  $x$  is  $f(x) = \frac{x}{1-x^2}$  not defined?

# ROUND TWO

IMTA 2008

1. The line with the equation  $y = ax + b$  passes through the points  $(-1, 0)$  and  $(0, -2)$ . Find the value of  $a$  and the value of  $b$ .

2. Find the equation of the tangent to the circle with the equation

$$(x-3)^2 + (y+2)^2 = 5 \text{ at the point } (2, -4).$$

Give your answer in the form  $ax + by + c = 0$ , where  $a, b$  and  $c \in N$ .

# ROUND THREE

IMTA 2008

1. Find the value of A and the value of B where

$$\frac{5}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}.$$

2. X is the matrix  $\begin{pmatrix} -2 & 4 \end{pmatrix}$  and Y is the matrix  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ .

Calculate the product XY.

# ROUND FOUR

IMTA 2008

1. Express  $\frac{8-i}{2+i}$  in the form  $a+bi$  where  $a$  and  $b \in \mathbb{Z}$  and  $i = \sqrt{-1}$ .

2. Given that  $2\sin^2 x + \cos^2 x = \frac{25}{16}$  and that  $x$  is an angle in the first quadrant find the value of  $\sin x$ .

## ROUND FIVE

IMTA 2008

1. In a TV debate on the health service, 6 people are chosen at random from a panel which consists of 5 nurses, 3 administrators and 4 consultants.

What is the probability that no consultant is chosen?

2. Find the value of  $x$  for which

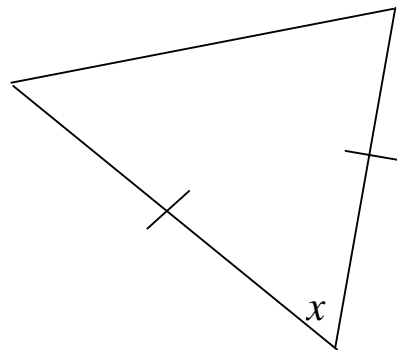
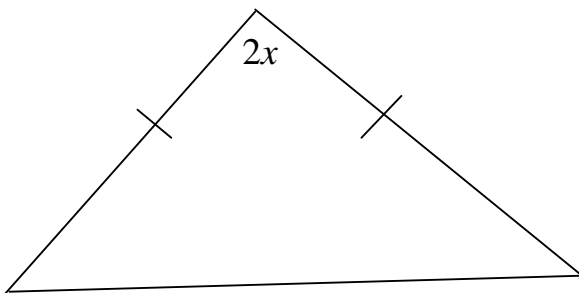
$$2 + \sqrt{x-2} = x-2, x \in R.$$

## ROUND SIX

IMTA 2008

1. Evaluate  $\frac{1}{\pi^2} \int_0^3 \frac{\tan^{-1} \frac{x}{3}}{x^2 + 9} dx.$

2.



The two triangles are equal in area and the marked sides are equal in length.

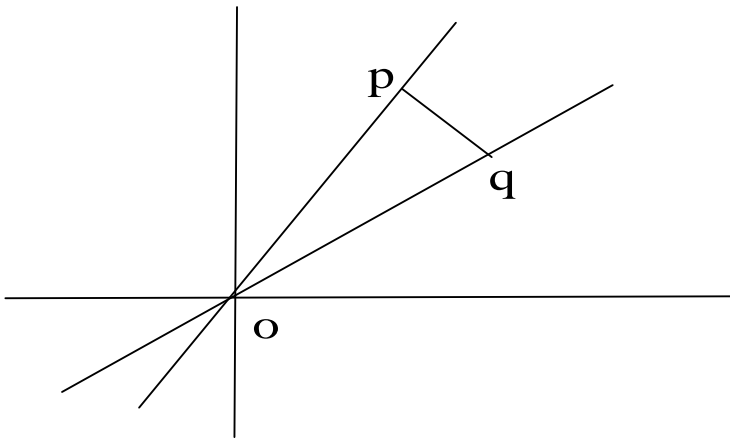
Calculate the value of  $x$ .

## ROUND SEVEN IMTA 2008

1. Given  $y = \sqrt{\frac{e^x}{1+e^x}}$  find the value of  $\frac{dy}{dx}$  for  $x = 0$ .

Answer in the form  $\frac{1}{a\sqrt{b}}$ , where  $a, b \in \mathbb{N}$ .

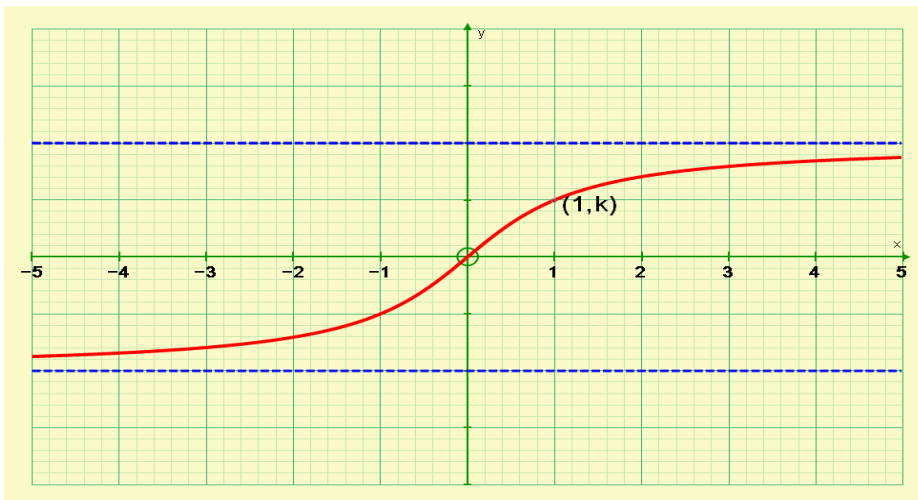
2. A bag contains 5 red markers and  $n$  blue markers. One marker is drawn and not replaced. A second marker is then drawn. If the probability that both are blue is one sixth how many blue markers are in the bag?
3. The slopes of the two lines  $oq$  and  $op$  are 1 and 2 respectively. Given that  $|op| = 1$ , and that  $pq \perp op$ , calculate the length of  $pq$ .



4. Find the equation of the tangent to the curve  $x^2y + xy^2 = 6$  at the point  $(2,1)$ .  
Give your answer in the form  $ax + by + c = 0$ , where  $a, b$  and  $c \in \mathbb{Z}$ .

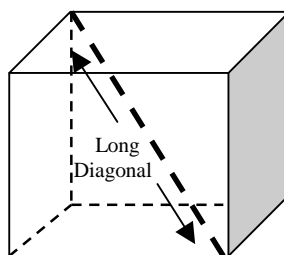
# ROUND EIGHT IMTA 2008

1. Find the real numbers  $p$  and  $q$  such that  $(p + iq)^2 = 15 - 8i$ .
2. A section of the graph of  $y = \tan^{-1} x$  is shown.



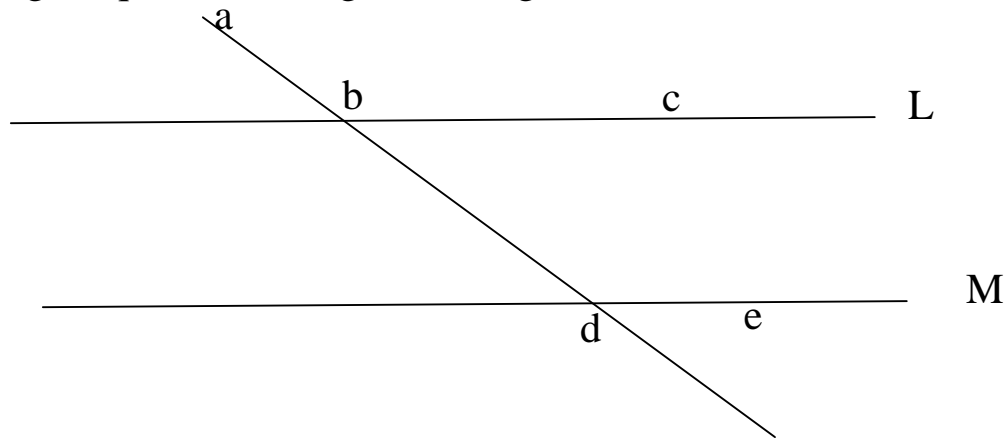
What is the value of  $k$ ?

3. Solve  $2e^{2y} - 3e^y - 2 = 0$ .
4. The volume of a solid cube in  $\text{cm}^3$  plus three times the total length of its edges in  $\text{cm}$  is equal to twice its surface area in  $\text{cm}^2$ . Calculate the length of its long diagonal in  $\text{cm}$ . Answer in the form  $a\sqrt{b}$ , where  $a$  and  $b \in \mathbb{N}$ .



# TIE BREAK IMTA 2008

1. Arrange  $7, 2\sqrt{11}, 4\sqrt{3}, 5\sqrt{2}, 3\sqrt{5}$ . in order, putting smallest number first.
2. Find the slope of the line joining  $(3, 5)$  and  $(-1, 2)$  in the form  $\frac{a}{b}$ ,  $(a, b \in N)$ .
3.  $y = \cos x \Rightarrow \frac{dy}{dx} =$
4.  $3x = 2y$  and  $xy = 24$ . Find a value for  $x$  and a value for  $y$ .
5.  $\frac{(n+1)!}{n!} =$
6. Name an angle equal to the angle  $\angle abc$  given that  $L \parallel M$ .



7. Factorise fully  $4x^2 - 16y^2$ .
8.  $(a - b)^3 =$
9.  $\frac{10!}{3!7!} =$
10.  $3 + 4 \times 5 =$